

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

**INFORMATION SUPERIORITY AND GAME THEORY:  
THE VALUE OF VARYING LEVELS OF INFORMATION**

by

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March 2002

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**INFORMATION SUPERIORITY AND GAME THEORY: THE VALUE OF  
VARYING LEVELS OF INFORMATION**

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## **ABSTRACT**

The ability to acquire and use information superiority to enhance combat power and contribute to the success of military operations is a primary factor in the fulfillment of the tenets of Joint Vision 2020. This thesis examines how various levels of information and information superiority affect strategy choices and decision-making in determining the payoff value for opposing forces in a classic zero-sum two-sided contest. The results show that if opposing forces possess options with equivalent strategic capabilities, the payoff advantage is determined by the quantity of choices from which to choose. The degree of advantage in payoff for the force with superior information is determined by the amount of choices and the quantity of bad information for the opponent. When a force possesses significantly fewer strategic options, more superior information is required to assume a payoff advantage, and for a force having more flexibility, significantly less information is required to affect an advantage in payoff. Additionally, we see that the effects of intelligence provides the greatest payoff advantage when a force possesses its maximum number of strategic options combined with the opposition also having its maximum number of choices.

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## **EXECUTIVE SUMMARY**

### **A. INTRODUCTION**

This thesis examines the effects of various levels of information superiority, and its effects on the advantages of military forces over their opponents. The research questions of this thesis address the potential benefits of the military's capability to use information as a force multiplier. The first research question addresses the value of the gain and loss of correct information that opposing forces possess and how it may affect potential outcomes of battle. The second research question examines the information value and affects to opposing forces if the quantity of courses of action available to opposing forces is varied. The third research question considers the value of the effects of intelligence on the decision-making of opposing forces in battle. The fourth research question addresses the effects of the value of information by varying the capabilities of opposing forces.

### **B. THE VALUE OF VARYING LEVELS OF INFORMATION**

The conceptual framework of the Armed Force's Joint Vision serves as the basis for focusing the strengths of each individual service component to exploit the full array of available capabilities. One of the most important underlying concepts of the Joint Vision is decision superiority and the value of information in defining force advantage, and in determining how decisions and choices of actions affect payoffs in battle. This thesis is a continuation of work performed for the U.S. Army by Dr. Jerome Bracken and Dr. Richard Darilek of RAND's Arroyo Center on the value of information. Bracken and Darilek explore the concepts of how much information superiority is necessary for U.S. military forces to obtain a quantifiable advantage over their opponents in the coming Information Age. Their research assumes that each force either possesses correct or incorrect information and that the knowledge of the decisions of an opponent were either known or unknown. This thesis extends their research by considering the value of information superiority, and its affects on the outcomes of decision-making as the information is varied between totally correct and incorrect, and as the simulated battle matrices are changed from symmetric to varying degrees of asymmetry.

Following Bracken and Darilek, we use Game Theory as a methodology to study the value of various forms of information in military operations. Specifically, zero-sum two-sided games are used, and each game includes both sides having three, five or ten courses of action available for achieving victory. In addition to varying the type and amount of information available to the two sides, the effects of opponents possessing different numbers of potential courses of action is also considered. Where Bracken and Darilek only use symmetric  $3 \times 3$ ,  $5 \times 5$  and  $10 \times 10$  matrices, this thesis simulates battles using  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 10$ ,  $10 \times 3$  and  $10 \times 5$  matrices in order to represent the probabilities of victory or payoffs for asymmetric forces having different amounts of strategies and choices from which to choose. The payoffs are generated using random numbers and are used to compute the averages of 1,000 trials for each battle simulation. Where Bracken and Darilek develop their payoffs using random numbers distributed uniformly between the boundaries of 0 and 100, this thesis simulates various levels of opposing force capabilities by using asymmetric payoff boundaries for the random number distributions.

The simulated game battles are coded in Excel and Crystal Ball. An example game matrix is shown below in Figure S.1:

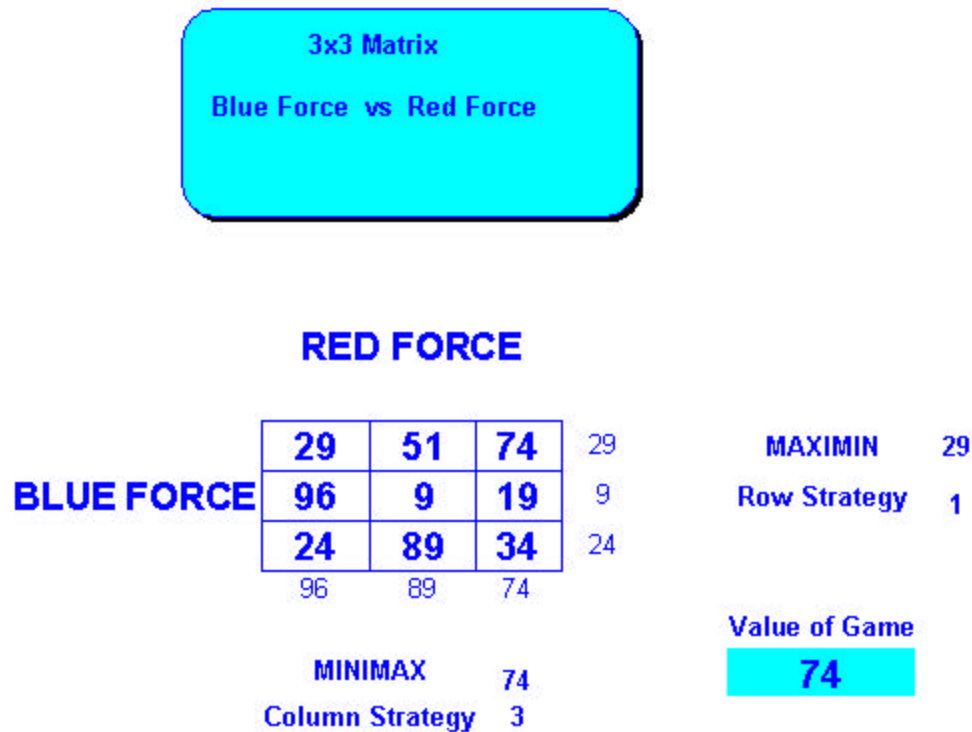


Figure S.1. Example 3x3 Game Matrix.

The above 3x3 matrix shows the Blue force choosing maximin row strategy 1 with a value of 29, and the Red force choosing minimax column strategy 3, for a value of 74. The payoff of the game battle is shown at the intersection of row 1 and column 3, for a value of 74. After 1,000 replications of simulated battle using numerous variations of the conditions for each force, the results are produced in the format of the following table and graph for each scenario:

Table S.1. Example Data for 3×3 Matrix With Red Force Degrading Info.

3x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	48.68	54.56	56.85	59.99
Median	48.00	55.00	56.00	61.00
Standard Deviation	20.03	21.72	23.30	23.71
Minimum	3.00	5.00	3.00	3.00
Maximum	98.00	99.00	99.00	99.00
Mean Std. Error	0.63	0.69	0.74	0.75

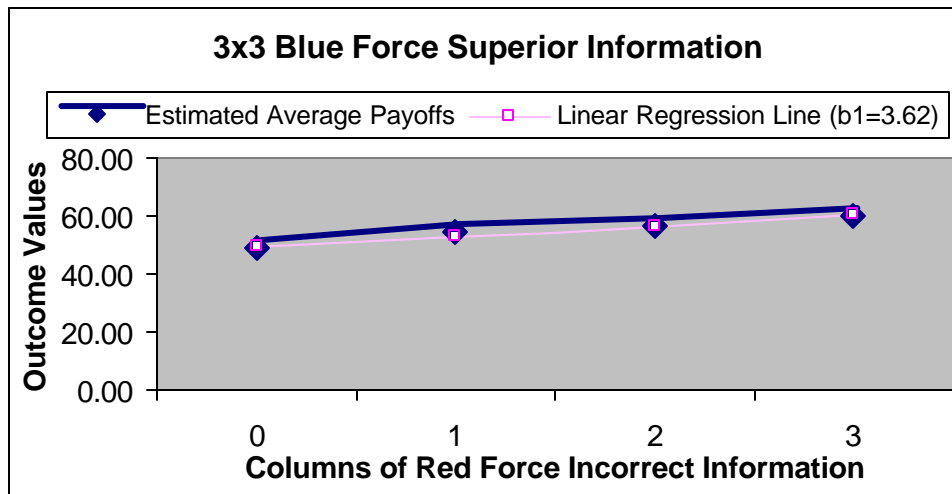


Figure S.2. Example Graph of Data for 3×3 Matrix With Red Force Degrading Information.

Table S.1 compares multiple measures on the estimated average payoffs as the amount of bad (meaning incorrect) information for the Red force increases from 0 bad columns to 3 bad columns. Figure S.2 shows how well the average payoffs align with a linear trend as the bad information increases.

Six sets of experiments are conducted and over 100,000 evaluations of matrix game simulations are performed in order to ensure that the estimated average payoffs are as accurate as possible over a variety of situations. The first experiment observes the effects of opposing forces in battle when possessing asymmetric strategy choices. The experiment demonstrates that if opposing forces possess options with equivalent strategic capabilities, the payoff advantage is determined by the quantity of choices from which to choose, and the payoff value increases linearly in favor of the force with the maximum number of choices. This suggests that flexible forces with more options have a

significant advantage even when they do not possess advantages in payoff or information superiority.

The second experiment addresses the value of varying levels of information superiority between asymmetric opposing forces. The degree of advantage in payoff for the force with superior information is determined by the amount of choices and the quantity of bad information. When a force possesses significantly fewer strategic options, more superior information is required to assume the payoff advantage. In addition, information tends to have a greater value and provide a larger payoff gain for less capable forces with fewer choices. For a force having more flexibility and more strategies, significantly less information is required to affect an advantage in payoff, and superior information is less valuable and produces a smaller marginal gain for more capable forces.

The third experiment considers the effects of intelligence on the payoffs of asymmetric opposing forces with common levels of information. The results of the experiment demonstrate that intelligence provides the greatest payoff advantage when a force possesses its maximum number of strategic options combined with the opposition also having its maximum number of choices. In the case where few options are available for opposing forces, intelligence provides minimal benefits to payoff advantage.

The fourth experiment examines the combined affects of both intelligence and information superiority, also known as information dominance, on the payoffs of opposing asymmetric forces. The results imply that on average, a force having information dominance produces a greater payoff gain when it has few strategies, and when its opposing force possesses significantly more capabilities.

The fifth experiment shows the effects on the payoffs of varying levels of superiority and inferiority in the capabilities of asymmetric forces. In the case of a force with increasingly superior capabilities, the use of the first superior option provides the largest payoff gain, and the benefits of additional good options level off thereafter. In fact, the first superior option provides the highest advantage to the force with the fewest choices, versus the most capable opposing force. The loss of force capability or the

increase in inferiority reduces the estimated average payoff most in the case when a force has its maximum number of choices and when its opponent possesses its minimum number of options. However, the more choices present with the threat of bad information, the more the protection exists against just a few bad strategies, whereas if few options are present, inferiority has a higher negative impact on each strategy loss.

The sixth experiment uses normal distribution payoffs to compare the estimated average outcome values to those of the uniform distribution. This test suggests that the conclusions may be robust to other symmetric payoff distributions.

The fundamental conclusion is that the benefits of various levels of information are dependent on numerous factors that affect a decision-maker's choice of strategy and ultimately the payoff of battle. These experiments reflect the effect of knowledge and capabilities on the likelihood of a successful outcome. It is the goal of this thesis to bring to the attention of the reader the level of influence that the control of information has on the determination and decisiveness of victory.

# **I. INTRODUCTION**

## **A. BACKGROUND**

### **1. Joint Vision 2010**

Joint Vision 2010 begins by addressing strategic changes and their implications for the Armed Forces in the near future. The focus of the document is to promote an environment and mindset that begins preparing for an uncertain future. One of the most important concepts stated in Joint Vision 2010 is that military leaders are in agreement that information superiority is among the most important enablers of victory. It is also agreed that an understanding of the value of information is key to ensuring the maximum effectiveness and capability of the Armed Forces.[Reference 18]

Information superiority is defined as the integration of offensive and defensive information operations; intelligence, surveillance, and reconnaissance, and other information-related activities that provide timely, accurate and relevant information; and command, control, communications, and computers activities that leverage friendly information systems.[Reference 18] For the year 2010, the goal of the Armed Forces is to improve the use of information by improving intelligence collection and assessment, modern information processing and command and control capabilities. By accomplishing this goal and achieving a state of information superiority, the military forces will be able to respond rapidly to any conflict in near real-time.

As a result of the military's improved capabilities to receive, process and disseminate information at an increasingly faster pace, day-to-day operations will be optimized with accurate, timely, and secure battlespace awareness information. Vital to battlespace awareness is the cooperative effect of intelligence support combined with the force commander's natural information assets.

The Department of Defense is developing a complementary command, control, communications, computer, intelligence, surveillance and reconnaissance (C<sup>4</sup>ISR) network architecture that will facilitate the development of revolutionary information and intelligence capabilities, similar to the private sector becoming increasingly interconnected through the worldwide growth of internet communications.[Reference 18]

The six principal components of the evolving C<sup>4</sup>ISR architecture for Joint Vision 2010 include:

- A robust multisensor information grid providing dominant awareness of the battlespace.
- A joint communications grid with adequate capacity, resilience and network management capabilities to rapidly pass relevant information to commanders and forces and to provide for their communications requirements.
- Advanced command and control processes that allow employment and sustainment of globally deployed forces faster and more flexibly than those of potential adversaries.
- A sensor-to-shooter grid to enable distributed joint forces to engage in coordinated targeting, cooperative engagement, integrated air defense and rapid battle damage assessment and dynamic follow-up strikes.
- An information defense capability to protect the globally distributed sensors, communications and processing networks from interference or exploitation by an adversary.
- An information operations capability to penetrate, manipulate or deny an adversary's battlespace awareness or unimpeded use of his own forces.

It is evident that our Armed Forces are truly dedicated to the achievement of information superiority and believe strongly in its advantages. The word “superiority” implies an advantage in one’s favor. In military operations, an advantage is transitory in nature and must be created and sustained through the conduct of training, exercises and operations.[Reference 18] Similarly, the attainment and maintenance of information superiority will require the same level of attention since its achievement is not an end in itself. Information superiority will only provide a competitive advantage when it is effectively translated into superior knowledge and decisions. The Armed Forces must be able to recognize and take advantage of superior information converted to superior knowledge in order to achieve “decision superiority” to enhance decision-making.[Reference 18]

Decision superiority is defined as a force’s advantage to unambiguously define certain choices of actions and certain outcomes.[Reference 5] Decision superiority results from superior information filtered through a commander’s experience, knowledge, training and judgment; the expertise of supporting staffs and other organizations; and the efficiency of associated processes. While changes in the information environment have



led to a majority of the focus on the contribution of information superiority to command and control, it is equally necessary to understand the complete realm of command and control and how it affects decision-making.

Command and control is most effective when decision superiority exists. Command and control is the exercise of authority and direction by force commanders over assigned and attached forces in the accomplishment of the mission.[Reference 18] Command and control includes planning, directing, coordinating and controlling forces and operations and is focused on the effective execution of the operational plan; but, the central function is decision-making.

In accordance with Joint Vision 2010, command and control will remain the primary integrating and coordinating function for operational capabilities and service components. As the nature of military operations evolves, there is a need to continually evaluate the nature of command and control organizations, mechanisms, systems and tools. The two major issues to address in this evaluation are command structures and processes, and the information systems and technologies that are best suited to support them. Encompassed within these two issues, examination of the following concepts and desired capabilities will serve as a catalyst for changes in doctrine, organization and training.

Information superiority is fundamental to the transformation of the operational capabilities of the joint force. Our forces will use superior information and knowledge to achieve decision superiority, to support advanced command and control capabilities and to reach the full potential of our military capabilities.

Joint Vision 2010, therefore, focuses and channels the entire Department's innovation, energy and resources towards a single long-term goal. The vision fully embraces the potential impact of information superiority and technological advances on military operations, resulting in a complete transformation of traditional warfighting concepts (e.g. maneuver, firepower, protection, sustainment) via changes in weapons systems, doctrine, culture and organization. This transformation is highly reliant on the employment of information and if executed properly will result in the success of four new operational concepts that together aim at achieving full-spectrum dominance: dominant

maneuver, precision engagement, full-dimensional protection and focused logistics.[Reference 18]

Dominant maneuver involves the multidimensional application of information, engagement and mobility which employs widely dispersed joint forces to apply decisive force upon an enemy's centers of gravity to compel an adversary to either react from a position of disadvantage or resign from the conflict. Dominant maneuver also involves the decisive application of force at critical points by leveraging U.S. asymmetric advantages to achieve operational objectives in minimum time and with minimum losses. The dominant maneuver concept requires several enhanced capabilities. One such capability is the ability to provide and process the required data in real-time. This will enable U.S. forces to be properly tailored for the specific operation, lighter and more rapidly deployable, and possess the requisite speed and force to mass effects and obtain positional advantages in time and space. Flexible, responsive logistics are critical to this concept. This tailor-to-task organizational ability, combined with focused logistics and advanced command and control, will reduce and disperse operational footprints and make it much more difficult for an adversary to fix and attack U.S. forces.

Precision engagement provides the means by which joint forces achieve desired effects across the spectrum of military operations. It promises the ability to find, fix, track and precisely target any military objective worldwide. Precision engagement leverages information superiority and global situational awareness through near real-time information on the objectives or targets, and a joint awareness of the battlespace for dynamic command and control. The result is a greater assurance of generating the desired effect against the objective or target, due to more precise delivery and increased survivability for all forces, weapons and platforms and the flexibility to rapidly assess the results of the engagement, then to reengage with precision when required.

The precision engagement concept transcends the notion of firepower. It encompasses achieving precise effects in cyberspace, as well as accurate and timely deliveries of humanitarian relief supplies or medical treatment to populations and directed psychological operations.

Protection for U.S. forces and facilities must be provided across the spectrum, from peacetime through crisis and at all levels of conflict. Achieving this goal requires a joint command and control architecture that is built upon information superiority and employs a full array of active and passive measures at multiple echelons. Full-dimensional protection will enable U.S. forces to safely maintain freedom of action, which is the freedom from attack and the freedom to attack. The development of a multi-tiered theater missile defense, combined with offensive capabilities to neutralize enemy systems before and immediately after launch, are prime examples of full-dimensional protection efforts. U.S. forces also need improved protection against chemical and biological weapons. New chemical and biological weapons detectors, improved individual protective gear, and a greater emphasis on collective protection are all critical to the Department's efforts to protect U.S. forces from these threats. Finally, full-dimensional protection includes defense against asymmetric attacks on information systems, infrastructure, and other critical areas vulnerable to nontraditional means of attack.

Focused logistics integrates information superiority and technological innovations to develop state-of-the-art logistics practices and doctrine. This will permit U.S. forces to accurately track and shift assets, thus facilitating the delivery of tailored logistics packages and more timely force sustainment. Focused logistics will streamline the logistics footprint necessary to support and sustain more agile combat forces that can be rapidly projected around the globe. Information intensive initiatives such as Automatic Identification Technology, Joint Total Asset Visibility, Global Transportation Network, and the Global Combat Support System will provide deployable, automated supply and maintenance information systems for precise and more responsive logistics.[Reference 18] These and other DoD-wide programs will be capable of supporting rapid unit deployment and employment. They will better support joint force commanders by eliminating redundant requisitions and reducing delays in the shipment of essential supplies.

It is clear that Joint Vision 2010 supports the idea that the operational effectiveness and mission capabilities of our Armed Forces are limited by the capacity of

its information infrastructure and the ability to enhance command, control and decision-making.

## **2. Joint Vision 2020**

Joint Vision 2020 extends the conceptual template established by Joint Vision 2010 in order to further guide the continuing transformation of the Armed Forces. In the year 2020, it is predicted that the nation will face even wider ranges of interests, opportunities and challenges and will require a military that can both win wars and contribute to peace.[Reference 19] If the Armed Forces are to be prepared for these challenges by being faster, more lethal and more precise, it must continue to invest in and develop new military capabilities. This vision describes the ongoing transformation to those new capabilities, and the extent to which the ability of our military to realize its full potential depends heavily upon our understanding of and performance in the information revolution.[Reference 19]

Information, information processing and communications networks are at the core of every military activity. The evolution of information technology will increasingly permit us to integrate the traditional forms of information operations with sophisticated all-source intelligence, surveillance and reconnaissance in a fully synchronized information campaign.[Reference 19] Joint Vision 2020 further emphasizes the commitment required to ensure that valuable information be provided on demand to warfighters, policy makers and support personnel in order to enhance combat power and contribute to the success of military operations.

The overarching focus of this vision is full spectrum dominance, achieved through the interdependent application of dominant maneuver, precision engagement, focused logistics and full dimensional protection. Attaining these goals requires the steady infusion of new technology and modernization. However, material superiority alone is not sufficient. Of greater importance is the development of doctrine, organizations, training and education, leaders and personnel that effectively take advantage of the technology.

The evolution of these elements over the next two decades will be strongly influenced by two factors. First, the continued development and proliferation of

information technologies will substantially change the conduct of military operations. These changes in the information environment make information superiority a key enabler of the transformation of the operational capabilities of the joint force and the evolution of joint command and control. Second, the U.S. Armed Forces will continue to rely on a capacity for intellectual and technical innovation. The pace of technological change, especially as it fuels changes in the strategic environment, will place a premium on our ability to foster innovation in our people and organizations across the entire range of joint operations. The overall vision of the capabilities we will require in 2020 rests on our assessment of the strategic context in which our forces will operate.

We will not necessarily sustain a wide technological advantage over our adversaries in all areas. Increased availability of commercial satellites, digital communications and the public internet, all give adversaries new capabilities at a relatively low cost. We should not expect opponents in 2020 to fight with strictly “industrial age” tools. Our advantage must, therefore, come from leaders, personnel, doctrine, organizations and training that enable us to take advantage of technology to achieve superior warfighting effectiveness.

Information operations are essential to achieving this full spectrum dominance. The joint force must be capable of conducting information operations, the purpose of which is to facilitate and protect U.S. decision-making processes, and in a conflict, degrade those of an adversary. While activities and capabilities employed to conduct information operations are traditional functions of military forces, the pace of change in the information environment dictates that we expand this view and explore broader information operations strategies and concepts. We must recognize that “nontraditional” adversaries who engage in “nontraditional” conflict are of particular importance in the information domain. The United States, itself, and U.S. forces around the world are subject to information attacks on a continuous basis regardless of the level and degree of engagement in other domains of operation. New offensive capabilities such as computer network attack techniques are evolving. Activities such as information assurance, computer network defense and counter-deception will defend decision-making processes by neutralizing an adversary’s perception management and intelligence collection efforts, as well as direct attacks on our information systems. Because the ultimate target of

information operations is the human decision-maker, the joint force commander will have difficulty accurately assessing the effects of those operations. This problem of “battle damage assessment” for information operations is difficult and must be explored through exercises and rigorous experimentation.

The continuing evolution of information operations and the global information environment holds two significant implications. First, operations within the information domain will become as important as those conducted in the domains of sea, land, air and space. Such operations will be inextricably linked to focused logistics, full dimensional protection, precision engagement and dominant maneuver, as well as joint command and control. At the same time, information operations may evolve into a separate mission area requiring appropriately designed organizations and trained specialists.

There also exists a significant potential for asymmetric engagements in the information domain. The United States has enjoyed a distinct technological advantage in the information environment and will likely continue to do so. However, as potential adversaries reap the benefits of the information revolution, the comparative advantage for the U.S. and its partners may become more difficult to maintain. As a result, our ever-increasing dependence on information processes, systems and technologies adds potential vulnerabilities that must be defended.

Joint Vision 2020 has a profound impact on the development of U.S. military capabilities. By describing those capabilities necessary to achieve success in 2020, three important concepts are established. First, JV 2020 established a common framework and language for the military forces to develop and explain their unique contributions to the joint force. Second, a process was created for conducting joint experimentation and training to test ideas against practice. Finally, a process began in order to manage the transformation of doctrine, organization, training, materiel, leadership and education, personnel and facilities necessary to make the vision a reality. Joint Vision 2020 builds on the foundation of Joint Vision 2010 and confirms the direction of the ongoing transformation of operational capabilities, and emphasizes the importance of further experimentation, exercises, analysis and conceptual thought, especially in the arenas of information operations.

### **3. Previous Research Conducted by Bracken and Darilek**

This thesis is a continuation of work previously performed for the U.S. Army by Dr. Jerome Bracken and Dr. Richard Darilek of RAND's Arroyo Center on the value of information. Bracken and Darilek contend that there are three overarching concepts that frame predictions about the future in which U.S. military forces are expected to operate.

The first concept is that "an Information Age is beginning to unfold and that it will largely define the first half of the twenty-first century." [Reference 1] It is believed that the Information Age will have the same relative impact as that of the Industrial Revolution during the latter half of the nineteenth century.

The dawning of the Information Age has given rise to advances in information technologies and information processing capabilities. The United States has led and maintains a significant advantage in the development of information-based technologies. This advantage is well grounded in U.S. military capabilities. The roots of the U.S. military's information-based technologies have been decades in the making, including the development and application of computer networks, precision-guided munitions, the Global Positioning System, and air and space based sensors. Yet, this rapid evolution in capabilities has not yet fundamentally transformed all of the essential elements of U.S. forces necessary to fully realize its maximum potential and effectiveness.

As information-based technologies and capabilities continue to mature, they become much less expensive, and can be rapidly incorporated by other military forces to enhance their capabilities. Just as in the past, the underlying information-based technologies upon which our future military will be based are becoming readily available to the military forces of many other nations. This underscores the imperative for the Department of Defense to develop a robust transformation strategy and mechanism to bring about the changes needed in the military's essential elements of strategy, doctrine, training, organization, equipment, operations, tactics and leadership in order to meet the challenges of the 21st century and the goals stated in Joint Vision 2010 and 2020.

Joint Vision 2010 identified technological innovation as a vital component of the transformation of our forces. Throughout the industrial age, the United States has relied upon its capacity for technological innovation to succeed in military operations, and the

need to do so will continue. It is important, however, to broaden our focus beyond technology and capture the importance of organizational and conceptual innovation as well. Innovation, in its simplest form, is the combination of new “things” with new “ways” to carry out tasks. In reality, it may result from fielding completely new things, or the imaginative recombination of old things in new ways. An effective innovation process requires continuous learning, a means of interaction and exchange that evaluates goals, operational lessons, exercises, experiments and simulations, accompanied by the inclusion of feedback mechanisms.

There exists, however, a high degree of uncertainty inherent in the pursuit of innovation. The key to coping with that uncertainty is bold leadership supported by as much information as possible. Leaders must assess the effectiveness of new ideas, the potential drawbacks to new concepts, the costs versus benefits of new technologies and the organizational implications of new capabilities. They must make these assessments in the context of an evolving analysis of the economic, political and technological factors of the anticipated security environment. Even though each of these assessments will have uncertainty associated with them, the best innovations have often come from people who made decisions and achieved success despite uncertainties and the lack of assurance of a positive outcome.

By creating and supporting innovation, the Armed Forces also create their best opportunities for coping with the increasing pace of change in the overall environment in which they function. Ultimately, the goal is to develop reasonable approaches with enough flexibility to recover from errors and unforeseen circumstances. There is no exact formula as to how the U.S. military should take advantage of the information revolution and the possibility of realizing its full potential. Rather, it requires extensive experimentation both to understand the potential contributions of emerging technologies and to develop innovative operational concepts to harness these new technologies.

The Information Age is predicted to transform the nature of future military operations to the extent of resulting in a Revolution in Military Affairs (RMA).[Reference 1] Bracken and Darilek’s second concept is “that for an RMA to occur, the role of information, its technologies and their organization is



critical.”[Reference 1] A Revolution in Military Affairs (RMA) occurs when a nation’s military seizes an opportunity to transform its strategy, military doctrine, training, education, organization, equipment, operations and tactics to achieve decisive military results in fundamentally new ways. History offers several such illustrations for example: the revolutionary French Republic’s levee en masse; the development of the blitzkrieg by the German Air Force and Army; and extensive, sustained, open ocean maritime operations developed by the U.S. Navy.[Reference 19] In all of these examples, the underlying technologies which made these revolutions possible were readily available to both opposing forces, however, in each case only one of the opposing forces made the commitment to transform the essential elements of its armed forces in such a manner as to achieve a dominant and decisive advantage in warfare. While exploiting the Revolution in Military Affairs is only one aspect of the Department of Defense’s transformation strategy, it is a crucial one. The refinement and expansion of the current RMA will provide the Department with a unique opportunity to transform the way in which it conducts the full range of military operations. Through the development of Information Age technologies, the RMA is expected to produce information superiority which future U.S. forces are expected to benefit from over their opponents.

The third concept, information superiority, is defined as “the capability to collect, process, and disseminate an uninterrupted flow of information while exploiting or denying an adversary’s ability to do the same.”[Reference 1] This third concept serves as the focus of this thesis. Bracken and Darilek explored the concepts of how much information superiority would be necessary for U.S. military forces to be able to obtain a quantifiable advantage over their opponents in the coming Information Age. This thesis extends their work by looking at: (1) asymmetric force, (2) varying levels of information superiority and intelligence and (3) different payoff distributions.

## **B. STATEMENT OF THESIS**

Before doing battle, in the temple one calculates and will win, because many calculations were made; before doing battle, in the temple one calculates and will not win, because few calculations were made; many calculations, victory, few calculations, no victory, then how much less so when no calculations? - Sun-Tzu: The Principles of Warfare “The Art of War”

Throughout history, the collection, exploitation and protection of information has been critical in command, control and intelligence. As discussed in Joint Vision 2010, the importance of information to the Armed Forces will continue to increase well into the future. What will differ, however, is the increased access to information and improvements in the speed and accuracy of prioritizing and transferring data brought about by advances in technology.[Reference 18] These technological advances generate numerous challenges for the U.S. military in achieving and maintaining information superiority over all potential enemies. In order to ensure our strategic upper hand, the understanding of the value of information and its applications to modern warfare are paramount. Joint Vision 2010 supports that information superiority will require a complete understanding of the value of information in order to successfully conduct both offensive and defensive information warfare.[Reference 18] Offensive information warfare will be conducted through the degradation or exploitation of the adversary's collection and use of information, and defensive information warfare will be conducted to protect our ability to perform information operations.[Reference 18]

Joint Vision 2020 places an even greater emphasis on the value of information and information superiority. Due to the fact that advances in information capabilities are proceeding so rapidly, there exists the risk of outstripping our ability to capture ideas, formulate operational concepts and develop the capacity to efficiently assess results.[Reference 19] The Armed Forces of the future will be required to take advantage of superior information by converting data into superior knowledge at an increasingly faster rate, in order to achieve the capability to formulate superior decisions. The ability to execute improved decision-making faster than an opponent can respond will increase a force's ability to shape, control and react to situations and changes in order to more efficiently accomplish objectives. Joint Vision 2020 states that the realization of the full potential of these changes requires not only technological improvements, but equally important, the continuing evolution of organizations and doctrine, and the development of relevant training to sustain a comparative advantage in the information environment.[Reference 19]

The research questions of this thesis address the military's ability to use information as a force multiplier. Consider a Blue force commander's level of

information of an impending Red force attack (e.g., type of attack, type of weapons used, size of force, etc.), and a Red force commander's level of information of the Blue force (e.g., location of camps, size of force, type of defenses, quality of communication systems, etc.). The first research question addresses the value of the gain and loss of correct information that opposing forces possess and how it may affect potential outcomes of battle.

The second research question examines the effects of the value of information to opposing forces if the quantity of decisions that are available to these forces are varied. For example, if the Blue force possesses more options, will this provide an advantage; or if the Red force has fewer choices to make, will this simplify their decision-making and allow their commander's to make better decisions?

The third research question considers the value of the effects of intelligence on the decision-making of opposing forces in battle. Consider the situation in which both the Blue and Red forces possess common knowledge of the values associated with their respective strategies, however, the Blue force knows what choices of strategy the Red force will decide before execution. It is examined how decision-making and deployment of forces is affected by the use of intelligence and information warfare.

The fourth research question addresses the effects of the value of information by varying the capabilities of opposing forces. The probabilities of victory and payoffs for each force is computed by calculating the averages of trials of games composed of random numbers using various distributions.

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## **II. GAME THEORY**

### **A. INTRODUCTION**

Game Theory, the methodology used in this thesis, is the study of the interactive behavior of decision-making. Game theorists are interested in situations in which a decision-maker's behavior affects not only their own gains and losses, but also those of opposing decision-makers. In order to analyze such interactive situations, game theorists use game theoretical concepts and mathematical tools to create simplified descriptions of real-life situations or "models".

The term "game" stems from the formal resemblance of these interactive decision problems to common parlor games such as Chess, Bridge, Poker, Monopoly, Diplomacy, Battleship, military strategy games such as the defense of targets against attack and economic games such as the price competition between two sellers.[Reference 5]

A game consists of a set of rules governing a competitive situation in which from two to the variable (n) individuals or groups of individuals choose strategies designed to maximize their winnings. The rules specify the possible actions for each player, the amount of information received by each as play progresses and the amounts won or lost in various situations.

For the social sciences, Game Theory is a powerful tool to analyze rational behavior, although Game Theory in general is not restricted to the analysis of rational actors. What makes Game Theory so attractive to social scientists is its ability to produce very general explanations of human and institutional behavior, which then can be applied to particular cases of human interaction.

In today's diversified and interdependent societies, scientific decision-making constitutes an essential part of military, political and economic processes. The consequences of seemingly simple decisions affect more and more people, and wrong decisions can lead to catastrophic outcomes, such as unemployment, environmental pollution, bankruptcies, international crises, social unrest and lost wars. Game Theory helps to understand why decision-makers make good or bad choices under different conditions, and how choices and choice processes can be improved.

## B. HISTORY

The mathematical theory of games was first developed by John Von Neumann and Oskar Morgenstern in their 1944 book Theory of Games and Economic Behavior. [Reference 11] Limitations in their mathematical framework initially made the theory applicable only under special and limited conditions. This situation has gradually changed over the past six decades, as the framework has been deepened and generalized. Refinements are still being made. However, since at least the late 1970s, it has been possible to say with confidence that Game Theory is an important and useful tool in an analyst's kit whenever confronted with problems in which one agent's rational decision-making depends on the expectations about what one or more other agents will do. Von Neumann and Morgenstern restricted their attention to zero-sum games in which no player can gain except at another's expense.

During the early 1950s, the work of John F. Nash further refined the developments of Von Neumann and Morgenstern. [Reference 11] Nash mathematically clarified the distinction between cooperative and non-cooperative games. In non-cooperative games, no outside authority assures that players stick to the same predetermined rules, and binding agreements are not feasible. Furthermore, he recognized that in non-cooperative games there exist sets of optimal strategies, called "Nash equilibria", used by players in a game such that no player can benefit by unilaterally changing his or her strategy if the strategies of the other players remain unchanged. [Reference 6] Because non-cooperative games are common in the real world, the discovery revolutionized game theory. Nash also recognized that such an equilibrium solution would also be optimal in cooperative games. He suggested approaching the study of cooperative games via their reduction to non-cooperative form and proposed a methodology, called "the Nash program", for doing so. [Reference 6] Nash also introduced the concept of "bargaining", in which two or more players collude to produce a situation where failure to collude would make each of them worse off. [Reference 11]

A major distinction between multi-person decision problems, Game Theory, and one-person decision problems is that in the one-person context, we are usually led to a well-defined optimization problem, like maximizing an objective function subject to some constraints. While this problem may be difficult to solve in practice, it involves no

conceptual issues. The meaning of "optimal decision" is clear; we must only find one. But in the interactive multi-person context, the very meaning of "optimal decision" is unclear, since in general, no one player completely controls the final outcome. This concept directly correlates to most military situations that involve a thinking adversary. In such cases, the payoff one player receives depends both on their actions and those of the opponent.

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### III. GAMING MODEL

#### A. DESCRIPTION OF ZERO-SUM, TWO-SIDED GAMES

Following Bracken and Darilek, this thesis uses Game Theory as a methodology to study the value of various forms of information in military operations. Specifically, zero-sum two-sided games are studied.

Each game is structured around potential military operations as depicted below:

		(Side 2) Red Force Strategies (j)			
		1	2	---	n
(Side 1) Blue Force Strategies (i)	1	$a_{1,1}$	$a_{1,2}$	---	$a_{1,n}$
	2	$a_{2,1}$	$a_{2,2}$	---	$a_{2,n}$
	.	.	.	.	.
	.	.	.	.	.
	m	$a_{m,1}$	$a_{m,2}$	---	$a_{m,n}$

Figure 3.1. Example Structure of Game Matrix.

The Blue and Red forces have strategy choices  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  respectively. For each pair of choices there exists a payoff  $a_{ij}$  generated using random numbers. The Blue force receives  $a_{ij}$  and the Red force loses  $a_{ij}$ . The Blue force therefore wishes to maximize the payoff, and the Red force wishes to minimize the payoff. This leads the Blue force to pursue what is referred to as a “maximin” strategy (to maximize their minimum possible payoff) and leads the Red force to pursue a “minimax” strategy (to minimize the maximum possible payoff).

Again, following Bracken and Darilek, the game theoretic strategies are computed by the game matrices given various conditions. The Blue force’s best strategy is determined by computing, for each possible choice (i), the worst (minimum) outcome  $a_{i,\min} = \min_j(a_{ij})$  that comes about if the Red force makes the best choice consistent with the Blue force’s choice of (i). The best choice for the Blue force is therefore the row

choice (i) that maximizes  $a_{i,\min} = \max_i(\min_j(a_{i,j}))$ . The Blue force chooses the row for which  $a_{i,\min}$  is the largest. The payoff for the Blue force is therefore at least as good as  $a_{\max,\min} = \max_i(\min_j(a_{i,j}))$ .

The Red force strategy is computed using the reverse process as that of the Blue force. For each of the possible choices (j) for the Red force, their least favorable strategy is the maximum outcome  $a_{\max,j} = \max_i(a_{i,j})$  that comes about if the Blue force makes best choice consistent with the Red force's choice of (j). The best choice for the Red force is therefore the column choice (j) that minimizes  $a_{\max,j} = \min_j(\max_i(a_{i,j}))$ . The Red force chooses the column for which  $a_{\max,j}$  is the smallest. The payoff for the Red force is, therefore, at least as good as (i.e., no higher than)  $a_{\min,\max} = \min_j(\max_i(a_{i,j}))$ . It is also important to note that like Bracken and Darilek, the game outcome values are determined using pure strategies (i.e., there is no random selection of rows and columns).

The conflict between the Blue force and the Red force is viewed abstractly as follows. In any given battle, the Blue force's choice of strategies has some effect on the outcome, as well as the choices of the Red force. Depending on the circumstances of the battle, however, these strategies make more or less of a difference. The value of information on the outcome of the conflicts is examined by considering vast arrays of battles in which strategies may have very different consequences on the outcome. The assessment is then made as to how much value information has on average over the array of battles. For each of the 1,000 different trials (battles), a payoff matrix is generated using random numbers with a specified distribution. The knowledge that each force has about the payoff matrix is varied, and the forces select strategies based on their level of knowledge.

For each game, the type of information available to the two sides, as well as the amount of information available is varied. Each game includes both sides having 3, 5, or 10 choices or strategies available for achieving victory. This is simulated using 3×3, 3×5, 3×10, 5×3, 5×5, 5×10, 10×3, 10×5 and 10×10 matrices in order to calculate the probabilities of victory or payoffs for Side 1 (the Blue force) versus Side 2 (the Red

force). The payoffs represent the calculated averages of 1,000 trials of the matrices games composed of random numbers. This ensures that the estimated average payoffs are close to the true values. Unless otherwise specified, the payoffs are drawn from a discrete uniform [0,100] random variable. For a uniform [0,100] random variable, the standard deviation is  $\sqrt{100^2/12} = 28.87$ . The mean of 1,000 such random variables has a standard error of  $28.87/(\sqrt{1000}) = 0.91$ . The standard error of the mean of 1,000 game values will be less than this, 0.91, boundary value.

The simulated game battles are coded in Excel and Crystal Ball. Example symmetric and asymmetric matrices are shown below:

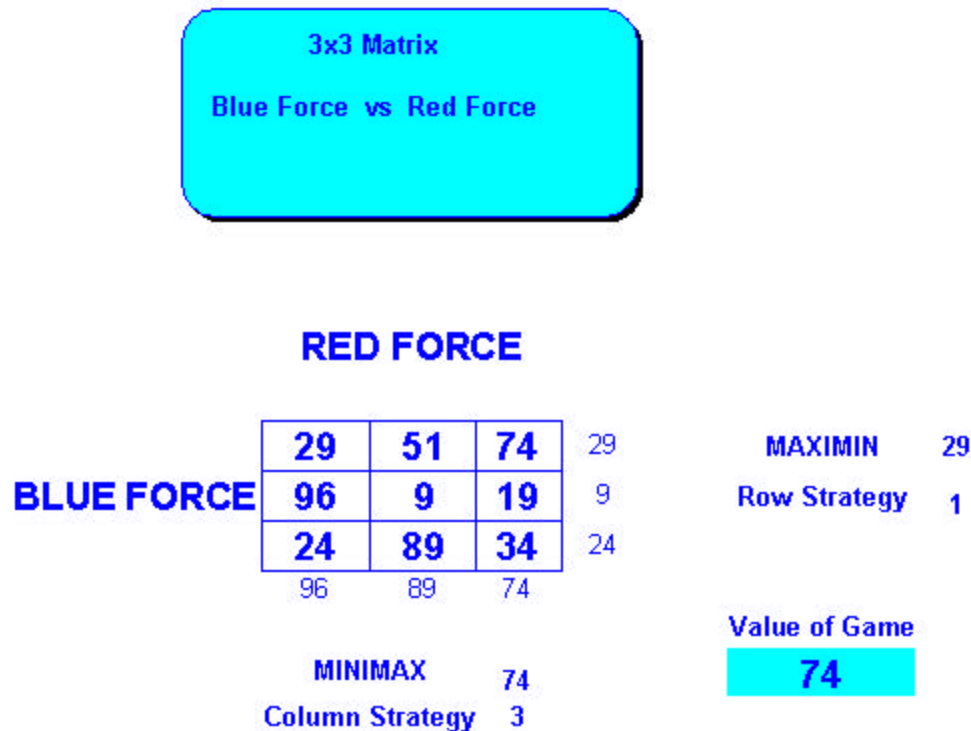


Figure 3.2. Example 3x3 Game Matrix.

The 3x3 symmetric matrix in figure 3.2 shows the Blue force choosing maximin row strategy 1 with a value of 29, and the Red force choosing minimax column strategy 3, for a value of 74. The payoff of the game battle is shown at the intersection of row 1 and column 3, for a value of 74. Similarly, figure 3.3 displays a 3x5 asymmetric matrix with the Blue force choosing maximin row strategy 3 for a value of 17, and the Red force

choosing minimax column strategy 1 for a value of 25. The payoff for this game battle, at the intersection of row strategy 3 and column strategy 1, is 17.

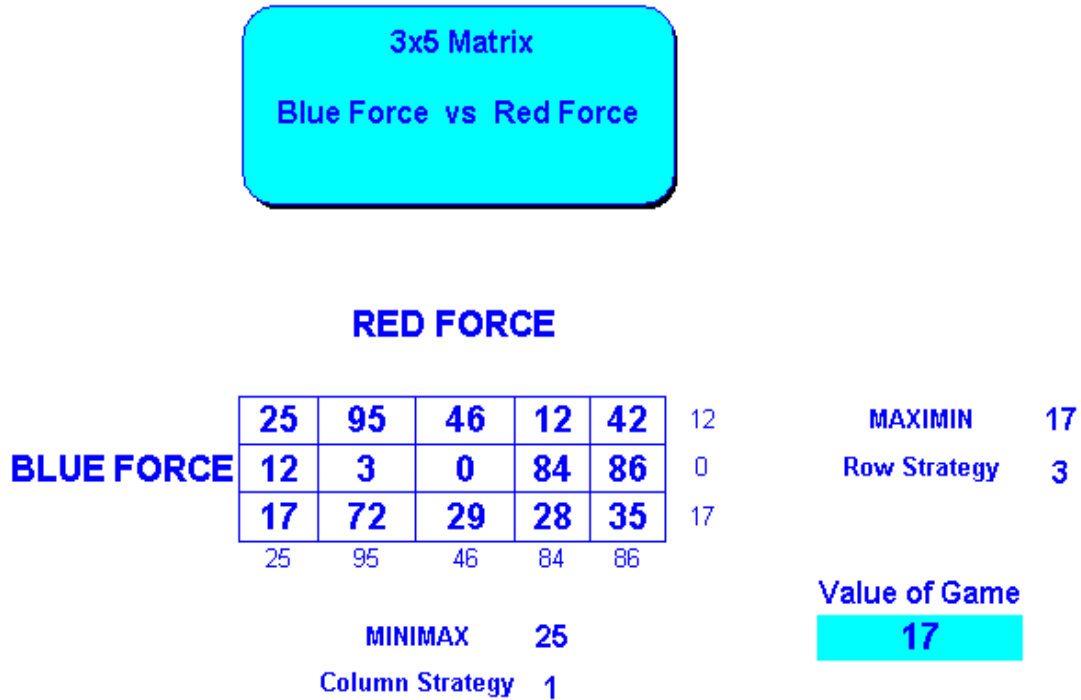


Figure 3.3. Example 3×5 Game Matrix.

## B. SUMMARY OF FINDINGS BY BRACKEN AND DARILEK

Table 3.1 shows a summary table of the findings by Bracken and Darilek for three sets of matrices for four games.[Reference 1] Bracken and Darilek use Game Theory as a means to quantify the value of information superiority.

Table 3.1. Data for Bracken and Darilek Experiments.

Number of Strategies Per Side	Game 1	Game 2	Game 3	Game 4
3×3	50.0	62.5	57.5	75.2
5×5	50.2	60.8	65.4	83.0
10×10	48.9	58.9	75.4	91.2

Bracken and Darilek observe three sizes of game matrices at four levels of information for the Blue force (Side 1). In the four games, the Blue force's information advantage increases as the games proceed from Game 1 to Game 4.

For the first game, where Side 1 and Side 2 both have common knowledge of all values of the payoff matrix, Side 1 chooses their maximin strategy and Side 2 chooses their minimax strategy. Since the underlying payoffs are random and distributed uniformly between 0 and 100, the expected payoff is 50. The numbers in Table 3.1 are the means of 1,000 replications. They conclude that since the expected payoff in Game 1 is always approximately 50 for all three symmetric ( $3 \times 3$ ,  $5 \times 5$  and  $10 \times 10$ ) matrices, the simulation is valid.

Secondly, Bracken and Darilek observe that the  $10 \times 10$  matrix (i.e., many options are available to both sides) provides the maximum increase in expected payoff from Game 1 to Game 4. For example, for Game 2 where Side 1 has correct information and Side 2 has incorrect information, the expected payoff yields a marginal gain of 8.9 over the expected payoff of 50.0. Game 3, with Side 1 knowing Side 2's move, yields a marginal gain of 25.4 over the expected payoff of 50.0. Game 4, with both bad information for Side 2 and Side 1 knowing Side 2's move, yields a marginal gain of 41.2 over the expected payoff of 50.0.

For the  $3 \times 3$  matrix, the results are quite different than for the  $10 \times 10$  case. Bracken and Darilek note that Game 2, where Side 2 has bad information, yields a marginal gain of 12.5 over the expected payoff of 50.0, compared with 8.9 in the  $10 \times 10$  case. Game 3, where Side 1 knows Side 2's move, yields a marginal gain of 7.5, compared with 25.4 in the  $10 \times 10$  case. This suggests that intelligence is more important when there are more choices an adversary can make. Game 4, where Side 2 has bad information and Side 1 knows Side 2's move, yields a marginal gain of 25.2, compared with 41.2 in the  $10 \times 10$  case.

It is determined, therefore, that the overall effects of information are not usually as great in the  $3 \times 3$  case (with fewer options) as in the  $10 \times 10$  case (with many options), with some exceptions. For example, bad information for Side 2 has more relative effect

in the  $3 \times 3$  case. Side 1 knowing Side 2's choice has more relative effect in the  $10 \times 10$  case. Combining the effects of Side 2 having bad information and Side 1 knowing Side 2's choice has a relatively more pronounced outcome in the  $3 \times 3$  case, than in the  $10 \times 10$  case.

The research conducted by Bracken and Darilek assumes that each force either possesses correct information or incorrect information and that the knowledge of the decisions of an opponent is either known or unknown. This thesis extends their research by considering the value of information superiority and its affects on the outcomes of decision-making, as the information varies between totally correct and incorrect, evaluated for asymmetric, as well as symmetric forces, and using different payoff distributions.

### **C. SPECIFICATION OF GAMES**

This section details the six experiments designed to represent, by analogy, different assumptions about the information available to two opposing forces, the joint U.S. Armed Forces (Blue force) versus an opponent (Red force). Additionally, varying levels of dimensionality are considered with respect to the number of courses of action available to both sides. The effects of information differ depending on these characteristics, which have an intuitive relationship with warfare.

#### **1. Replication of Bracken and Darilek Experiment of Common and Correct Knowledge With Varying Strategy Choices**

The first game simulation is a replication of the experiment performed by Bracken and Darilek simulating opposing forces with common and correct knowledge. This thesis extends their experiment by varying the number of strategies available for each force. Whereas Bracken and Darilek use only  $3 \times 3$ ,  $5 \times 5$  and  $10 \times 10$  symmetric matrices, this thesis varies the number of strategies and simulates additional combinations including  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 10$ ,  $10 \times 3$  and  $10 \times 5$  matrices. A more capable force will likely have more options.

#### **2. Value of Various Levels of Correct and Incorrect Information**

In this experiment, both forces initially know and agree on the random numbers that appear in all rows and columns of the  $3 \times 3$ ,  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 5$ ,  $5 \times 10$ ,  $10 \times 3$ ,  $10 \times 5$  and  $10 \times 10$  payoff matrices created by their opposing strategies. Therefore, both sides

have the same type of information, and neither benefits qualitatively from information superiority. However, the quantity of this information for the Red forces is decremented from 100% to 0%. It is assumed that the Red force possesses information degradation (i.e., the Blue force gains the advantage of information superiority via offensive information warfare or a lack of information systems by the Red force). The information degradation by the Red force is simulated by replacing correct information with incorrect information, one column at a time, from payoffs using a separate matrix with a different set of random numbers using the same distribution as that of the correct matrix. These results are compared to those of Bracken and Darilek, who found that when neither side enjoys information superiority, the contribution of knowledge to winning has the same value for both forces.[Reference 1] They also found that this seems to hold regardless of the number of strategies available to each side in their symmetric games.[Reference 1] It is determined, however, that if one side possesses bad information, there exists a marginal gain for the force with correct information, and this gain is more prominent for the smaller  $3 \times 3$  matrix, as opposed to a smaller gain for the larger matrix of  $10 \times 10$ .

### **3. Replication of Bracken and Darilek Experiment of Common and Correct Knowledge and Intelligence With Varying Strategy Choices**

For this experiment, both forces initially have correct knowledge of the values associated with their strategies in the  $3 \times 3$ ,  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 5$ ,  $5 \times 10$ ,  $10 \times 3$ ,  $10 \times 5$  and  $10 \times 10$  matrices. As a result, the best strategy for the Red force is to make a decision using minimax. In this game, however, the Blue force knows the Red force's strategy with certain probability and has knowledge of the decisions that the Red force makes. The Blue force is therefore using reconnaissance and or intelligence to obtain information about the Red force's strategic intentions. This allows the Blue force to focus only on the payoffs corresponding to the minimax choice of the Red force vice using the maximin strategy. The Blue force, therefore, is able to maximize their payoff given the Red force's course of action. These results are compared to those of Bracken and Darilek, who found that with similar levels of information, one side knowing the moves of the opponent, yields a greater marginal gain in payoff for the force with the intelligence and that this gain is more prominent for the larger matrix of  $10 \times 10$ , as compared to the smaller matrices of  $3 \times 3$  and  $5 \times 5$ .

#### **4. Value of Various Levels of Information With Intelligence**

In this experiment both forces initially have correct information, thereby knowing the values associated with the strategies of their opponent. Additionally, the Blue force knows the Red force's choice of strategy. In this case, the Blue force possesses both correct information and intelligence. The Red force's level of correct information is varied by degrading it from 100% to 0% for each matrix. The Red force is therefore assumed to possess information degradation as a result of offensive information warfare and deception tactics employed by the Blue force. These results are compared to those of Bracken and Darilek, who found that with one force having bad information and the opposing force having both correct information and prior knowledge of the choices to be made by their opponent, these criteria yield the highest marginal gain of payoff among all scenarios. It is also noted that this increase of payoff becomes more prominent as the number of choices increases.

#### **5. Value of Varying Capabilities of Forces**

This experiment addresses the affects of various levels of information on opposing forces as the capabilities of the forces are altered. This scenario varies the capabilities of opposing forces by using different boundaries for the random number distributions. For each payoff value in the matrices, the Blue force first has increasingly superior force capabilities. Superior forces are simulated by altering the uniform distribution of the Blue force row choices from 50 to 100. Similarly, the Blue force simulates increasingly inferior force capabilities by altering the boundaries of the uniform distribution of the row choices from 0 to 50. The varying levels of superior and inferior capabilities for the Blue force are conducted by increasing the number of different rows for the Blue force from 0% to 100% for both superior and inferior forces.

#### **6. Value of Varying Levels of Information Using Normal Distribution Payoffs**

A variation of the methodology used in this thesis is to generate payoffs in each matrix using different distributions. This differs from the experiments conducted by Bracken and Darilek who generate payoffs for matrices using random numbers uniformly distributed between 0 to 100. In this experiment, the matrices are developed with random numbers using the truncated normal distribution with a mean of 50, a standard deviation



of  $\sqrt{100^2/12} = 28.87$ , a minimum of 0 and a maximum of 100. These normal random variables have the same mean and variance as the uniform random variables.

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## IV. DATA AND ANALYSIS

This section displays the statistical results (mean, median, standard deviation, minimum, maximum and mean standard error) for each of the six experiments in a table format followed by a graph and an analysis of the estimated average payoffs for each corresponding matrix scenario. Higher average payoffs reflect better outcomes for the Blue force. When the two forces are equivalent, the average will be close to 50 (allowing for some random variation).

### A. REPLICATION OF BRACKEN AND DARILEK EXPERIMENT OF COMMON AND CORRECT KNOWLEDGE WITH VARYING STRATEGY CHOICES

#### 1. Introduction

In this test, the first experiment performed by Bracken and Darilek is replicated by simulating opposing forces with common and correct knowledge. Their experiment is extended by varying the amount of strategies available for each force. Whereas the experiment conducted by Bracken and Darilek only uses symmetric  $3 \times 3$ ,  $5 \times 5$  and  $10 \times 10$  matrices, this experiment varies the number of strategies and simulates all combinations including asymmetric  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 10$ ,  $10 \times 3$  and  $10 \times 5$  matrices. The intent of this experiment is to observe the effects of different numbers of strategies on opposing forces with equal levels of common and correct information.

## 2. Analysis of Data and Graph

Table 4.1. Data for Both Forces With Correct Information.

Statistics	3x3	3x5	3x10	5x3	5x5	5x10	10x3	10x5	10x10
Mean	48.93	41.11	30.28	58.32	49.02	40.21	68.65	59.72	50.16
Median	48.00	40.00	29.00	58.00	49.00	39.00	69.00	60.00	50.00
Standard Deviation	19.81	18.10	16.34	17.47	18.27	18.19	15.09	17.73	19.18
Minimum	4.00	2.00	0.00	10.00	6.00	3.00	24.00	17.00	0.00
Maximum	93.00	89.00	98.00	96.00	94.00	98.00	97.00	98.00	96.00
Mean Std. Error	0.63	0.57	0.52	0.55	0.58	0.58	0.48	0.56	0.61

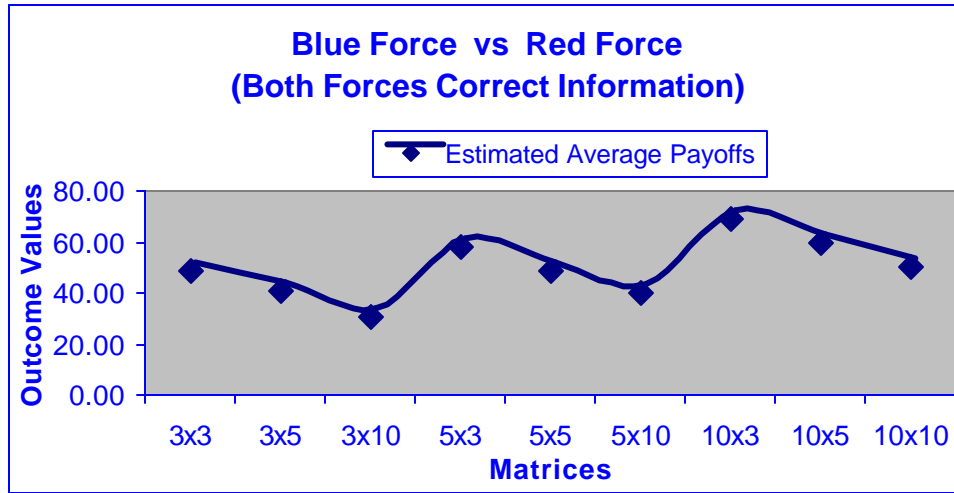


Figure 4.1. Graph of Data for Both Forces With Correct Information.

The statistics in Table 4.1 display the mean, median, standard deviation, minimum, maximum and mean standard error of the outcome values for each of the 3x3, 3x5, 3x10, 5x3, 5x5, 5x10, 10x3, 10x5 and 10x10 payoff matrices that simulate battle between the Blue and Red forces. Figure 4.1 graphs the mean outcome values in Table 4.1, versus the corresponding payoff matrices. For example, the graph depicts a negative slope for the 3x3 to 3x10 payoff matrices, as the mean outcome value decreases in favor of the Red force from 48.93 to 30.28. Similarly, the graph further shows the mean outcome value decrease from 58.32 to 40.21 for the 5x3 to 5x10 payoff matrices and from 68.65 to 50.16 for the 10x3 to 10x10 payoff matrices.

As the Blue force's number of strategic options increase from 3 to 5 and 10, while the Red force's number of choices remain constant at 3, the payoff value increases in

favor of the Blue force by a value of 19.72. Similarly, when the Red force's number of choices of strategy are held constant at 5 and the Blue force's options vary from 3 to 5 and 10, the margin of victory for the Blue force increases by a slightly lower value of 18.61. And finally, when the Red force's number of strategies are held constant at 10, while the Blue force's choices vary from 3 to 5 and 10, the Blue force's advantage increases by an outcome value of 19.88. This suggests that flexible forces (i.e., those with several options) have a significant advantage even when they have no information or payoff advantage. The results of this experiment draw comparisons to the fundamentals of the Joint Vision dominant maneuver concept in that U.S. forces are trained to seize the advantage in conflict by employing widely dispersed, properly tailored, lighter and rapidly deployable joint forces that strike at an enemy's critical points and centers of gravity.

## **B. VALUE OF VARIOUS LEVELS OF CORRECT AND INCORRECT INFORMATION**

### **1. Introduction**

For this game, both forces initially have correct information, meaning that both sides know the values of the payoffs in each matrix before making a decision. The quality of the information for the Red force is then altered by replacing perfect information in the options of the Red force with incorrect information (bad columns) from a separate matrix generated independently with the same distribution. For example, for the 3x3 matrix, the first column is replaced with incorrect information (one bad column) and the battle is simulated. Next, the matrix is simulated with two bad columns, and finally with all three columns of incorrect information. In the latter case, the Red force possesses 100% bad information, and the Blue force assumes information superiority.

This experiment addresses the value of one force gaining information superiority over its opposition. Each column in the data tables represents the payoffs of the matrices, as the number of bad columns of incorrect information for the Red force is increased from 0% to 100% column by column for each matrix.

## 2. Analysis of Data and Graphs

Table 4.2. Data for 3×3 Matrix With Red Force Degrading Information.

3x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	48.68	54.56	56.85	59.99
Median	48.00	55.00	56.00	61.00
Standard Deviation	20.03	21.72	23.30	23.71
Minimum	3.00	5.00	3.00	3.00
Maximum	98.00	99.00	99.00	99.00
Mean Std. Error	0.63	0.69	0.74	0.75

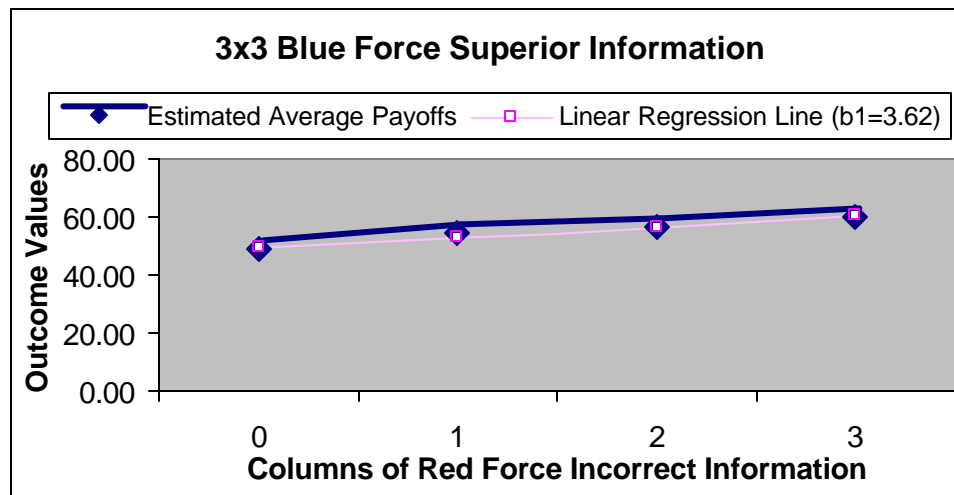


Figure 4.2. Graph of Data for 3×3 Matrix With Red Force Degrading Information.

For the symmetric 3×3 payoff matrix, as the quantity of incorrect column strategies for the Red force increases from 0 to 3, the battle outcome value increases in favor of the Blue force for a total marginal gain of 11.31. The largest increase in payoff for the Blue force occurs when the Red force chooses among 1 of 3 incorrect courses of action.

In Figure 4.2, the estimated average payoffs line suggests that a strong locally linear relationship exists between the independent variable (columns of Red force bad information) and the dependent variable (payoffs). Since the standard deviations are all about the same, ordinary least squares is used to determine the local underlying trend between the payoff values and the loss of information for the Red force. As a result, the linear regression equation (average payoff = 49.582 + 3.622 \* Red force bad columns)

fits the data very well and describes the relationship between payoffs and Red force bad information. From the regression model, we see that for every bad Red force column, the estimated average payoff increases in favor of the Blue force by a value of 3.622. In Figure 4.2, the estimated slope ( $b_1$ ) is shown by ( $b_1 = 3.62$ ). This notation is used on the upcoming figures in which a linear regression fits the data. These results suggest that when a force has a significant disadvantage, there is continual and substantial added value to information superiority.

As will be seen in the forthcoming figures, in all of our models, the regressions fit very well, typically with an  $R^2$  of greater than 0.90. When a linear fit is sufficient, the slope ( $b_1$ ) is shown. When a non-linear fit is required (e.g., the data align more along a quadratic or cubic curve), this non-linearity is noted. All of the regressions fit in this thesis are performed by regressing the average payoff against the levels of bad information. For example, the regression line in Figure 4.2 is generated using the four points in the figure and not the 4,000 observations that went into calculating the means (this is known as ecological regression). Either approach yields the same regression equation, which is our goal. It should be noted, however, that a consequence of regressing on the average payoffs, rather than the raw numbers, is that the  $R^2$  values in the regression averages are much higher than they otherwise would be. In all of the regression models, we determine whether a linear fit is sufficient, and if so, what the estimated slope ( $b_1$ ) is, or whether there exists a nonlinear (e.g., quadratic or cubic) relationship. The reader is cautioned that the regressions apply only to the regions used to develop the equations and cannot be extrapolated beyond the data considered or to other models. In fact, doing so may yield nonsensical results. For example, extrapolating the fits can result in expected average payoffs of greater than 100, which is not feasible.

Table 4.3. Data for 3×5 Matrix With Red Force Degrading Information.

3x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	41.28	43.43	47.03	51.82	54.59	57.76
Median	41.00	42.00	45.00	49.00	53.00	58.00
Standard Deviation	18.42	20.56	23.01	25.07	25.19	25.07
Minimum	2.00	3.00	2.00	1.00	5.00	2.00
Maximum	91.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.58	0.65	0.73	0.79	0.80	0.79

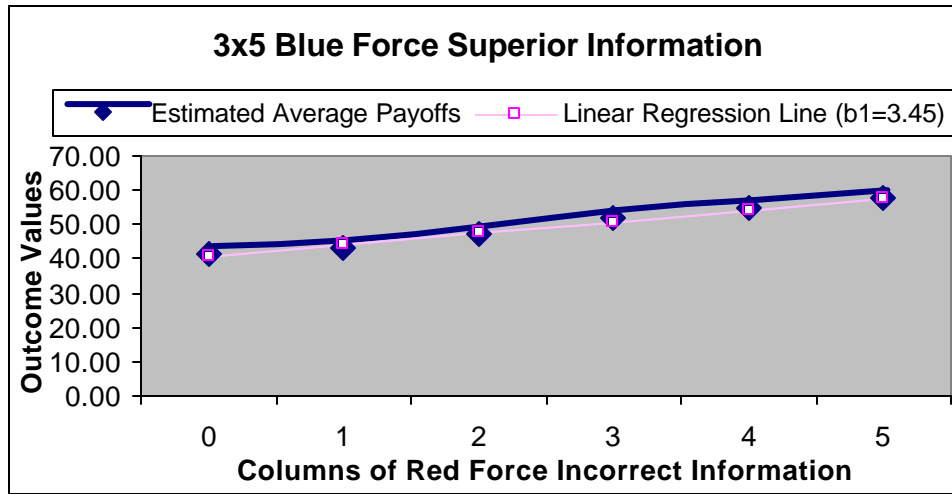


Figure 4.3. Graph of Data for 3×5 Matrix With Red Force Degrading Information.

In the 3×5 case, the Red force, being a more capable force with more options, has the advantage when both forces possess correct information, however, as the quantity of incorrect information for the Red force increases from 0 column strategies to 5, the payoff increases in favor of the Blue force (from 41.28 for 0 bad strategies to 57.76 for all 5 bad strategies) for a gain of 16.48. The Blue force, in this case, needs to degrade at least 3 of the Red force's strategies to overcome the disadvantage (i.e., average payoff greater than 50). These results show the benefit of information superiority for the Blue force in overcoming its force disadvantage to the Red force.



Table 4.4. Data for 3×10 Matrix With Red Force Degrading Information.

3x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	30.48	32.87	34.94	37.57	39.71	41.90	43.92	47.92	49.65	51.78	52.84
Median	29.00	31.00	32.00	33.00	36.00	37.00	40.00	45.00	48.00	49.00	53.00
Standard Deviation	15.18	18.35	20.13	22.71	24.54	24.74	26.09	26.36	26.66	27.02	26.84
Minimum	1.00	2.00	2.00	2.00	1.00	1.00	2.00	0.00	2.00	1.00	1.00
Maximum	77.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.48	0.58	0.64	0.72	0.78	0.78	0.82	0.83	0.84	0.85	0.85

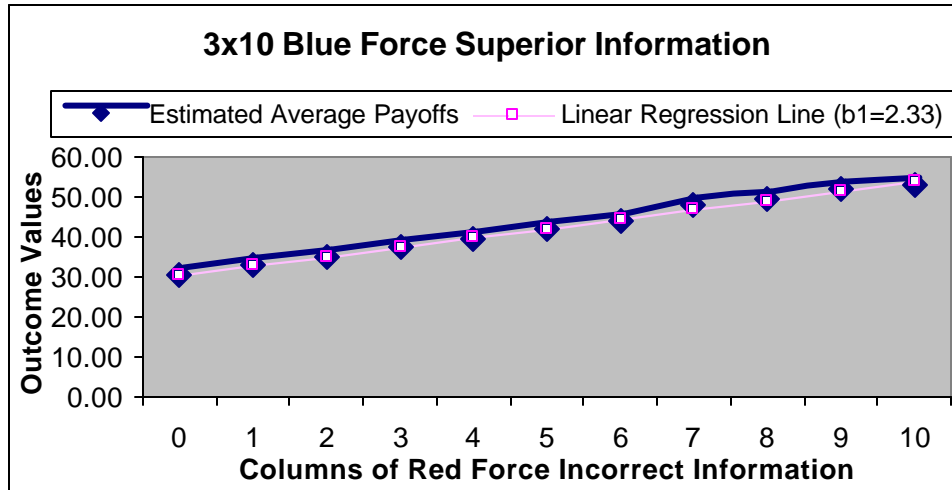


Figure 4.4. Graph of Data for 3×10 Matrix With Red Force Degrading Information.

In the 3×10 case, the Red force has an even greater advantage with 10 options to the Blue force's 3, however, as the quantity of poor strategies for the Red force increases from 0 to 10, the outcome value increases linearly in favor of the Blue force (from 30.48 for 0 bad strategies to 52.84 for all 10 bad strategies) for a total marginal gain of 22.36 for the Blue force. In this case, the Blue force requires that the Red force be subjected to as many as 9 bad column strategies before the Blue force is able to assume the advantage and overcome the Red force's initial dominance. These results show the significant benefits of superior information to forces that have large disadvantages.

Table 4.5. Data for 5×3 Matrix With Red Force Degrading Information.

5x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	58.98	62.35	63.81	66.47
Median	59.00	63.00	65.00	67.00
Standard Deviation	18.43	18.75	20.71	20.82
Minimum	6.00	13.00	10.00	10.00
Maximum	97.00	99.00	99.00	99.00
Mean Std. Error	0.58	0.59	0.65	0.66

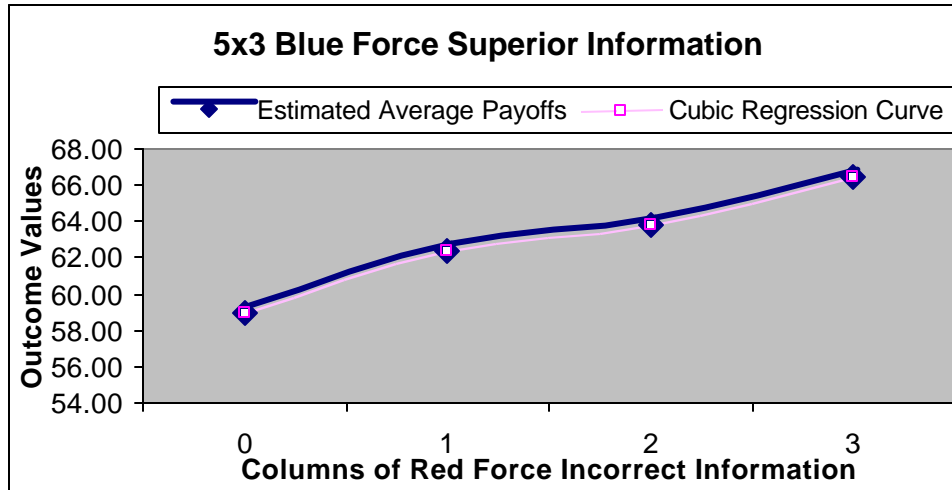


Figure 4.5. Graph of Data for 5×3 Matrix With Red Force Degrading Information.

In the 5×3 case, the Blue force is the more capable force with slightly more strategy choices when both forces possess correct information, however, as the quantity of incorrect information for the Red force increases from 0 column strategies to 3, the Blue force's advantage increases even more (from 58.98 for 0 bad Red force strategies to 66.47 for all 3 bad Red force strategies) for a small increase in the estimated average payoff battle of 7.49. These results show that since the Blue force has the advantage, the benefits of information superiority are small and a cubic relationship exists between estimated average payoffs and Red force bad information. In Figure 4.5, a smaller scale (54 to 68) is used for the outcome values since the range of the estimated average payoff is less than 10.

Table 4.6. Data for 5×5 Matrix With Red Force Degrading Information.

5x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	50.49	51.87	54.18	56.09	57.60	60.15
Median	51.00	51.00	54.00	56.00	58.00	61.00
Standard Deviation	18.50	20.68	20.88	21.65	22.89	23.20
Minimum	6.00	8.00	6.00	6.00	4.00	6.00
Maximum	91.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.58	0.65	0.66	0.68	0.72	0.73

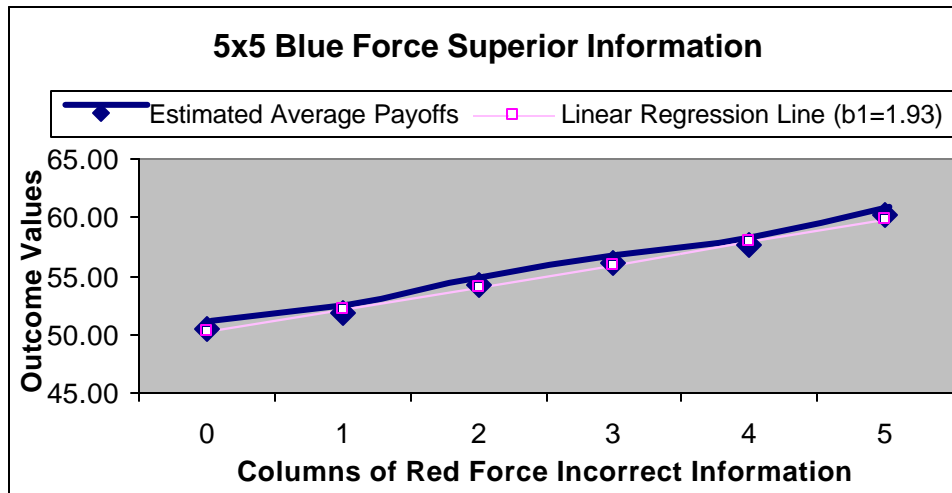


Figure 4.6. Graph of Data for 5×5 Matrix With Red Force Degrading Information.

The results of the 5×5 symmetric payoff matrix are similar to the findings for the 3×3 case (see Table 4.2). Each opposing force possesses the same number of strategies, therefore providing no advantage in force capability. As the quantity of incorrect information for the Red force increases from 0 column strategies to 5, the Blue force's advantage increases (from 50.49 for 0 bad Red force strategies to 60.15 for all 5 bad Red force strategies) for an increase in estimated average payoff of 9.69 for the Blue force, as compared to an increase from 48.66 to 59.99 for a marginal gain of 11.31 for the Blue force in the 3×3 matrix. In Figure 4.6, the scale is reduced to (45 to 65) for the outcome values since the range of the estimated average payoff is less than 10.

Table 4.7. Data for 5×10 Matrix With Red Force Degrading Information.

5x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	39.70	40.76	42.50	44.63	45.77	47.18	48.93	50.76	53.67	54.87	56.07
Median	39.00	40.00	41.00	43.00	44.00	46.00	46.00	49.00	54.00	55.00	57.00
Standard Deviation	18.06	18.87	20.47	21.75	22.67	23.46	24.13	24.67	25.97	25.62	25.55
Minimum	2.00	2.00	3.00	5.00	4.00	5.00	4.00	4.00	3.00	4.00	5.00
Maximum	91.00	98.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.57	0.60	0.65	0.69	0.72	0.74	0.76	0.78	0.82	0.81	0.81

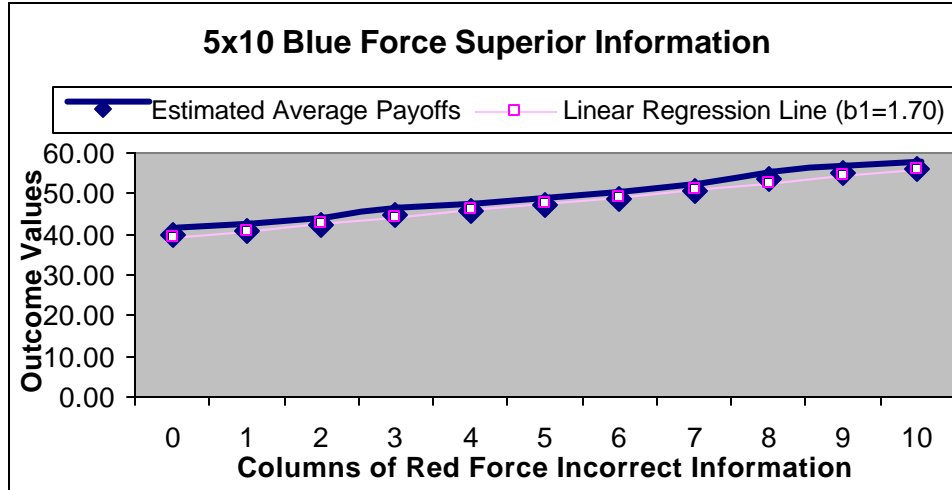


Figure 4.7. Graph of Data for 5×10 Matrix With Red Force Degrading Information.

For the 5×10 payoff matrix, the Red force possesses more strategic options than the Blue force and is therefore more capable. As the quality of all 10 of the column strategies is reduced, the advantage in battle is shifted to the Blue force by a value of 16.37. The Blue force assumes the advantage, when the Red force suffers a loss of 7 correct courses of action. The Blue force's linear increase in battle outcome advantage for the 5×10 scenario of 16.37 is slightly less than its advantage in the 3×10 case for a value of 22.36 (see Table 4.4). We also see that the slopes decrease, as the Blue force loses its advantage, from the 5×3 case to the 5×5 and 5×10 matrices. These results suggest that the benefits of information superiority are greater in the case when a force possesses more of a disadvantage, as in the 3×10 matrix as opposed to the 5×10 matrix.

Table 4.8. Data for 10×3 Matrix With Red Force Degrading Information.

10x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	68.65	70.66	71.64	72.38
Median	70.00	72.00	73.00	74.00
Standard Deviation	15.82	16.04	16.69	17.37
Minimum	17.00	27.00	20.00	26.00
Maximum	98.00	99.00	99.00	99.00
Mean Std. Error	0.50	0.51	0.53	0.55

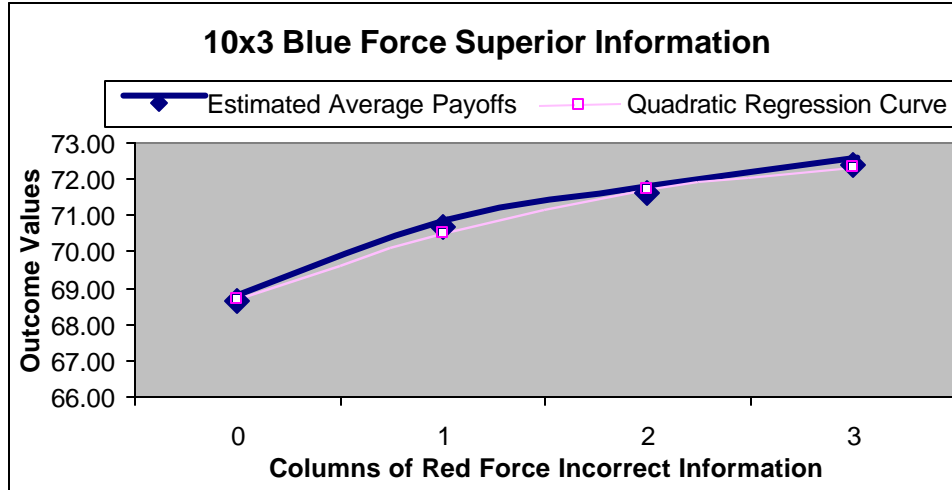


Figure 4.8. Graph of Data for 10×3 Matrix With Red Force Degrading Information.

In the 10×3 case, the Blue force is even more capable over its opponent than in the 5×3 payoff matrix (see Table 4.5), by possessing twice as many strategic options. However, as the quantity of incorrect information for the Red force increases from 0 column strategies to 3, the Blue force's advantage of 3.73 in the 10×3 case increases less than that of the 5×3 matrix for a value of 7.49. In Figure 4.8, the scale is reduced to (66 to 73) for the payoff outcome values since the range of the payoff is less than 10. The estimated averages payoffs and the increasing rows of superiority show a quadratic relationship. Therefore, the quadratic term in the equation is statistically significant at the 0.10 level. These results suggest that when a force has a significant advantage, there may be little added benefit from information superiority.

Table 4.9. Data for 10×5 Matrix With Red Force Degrading Information.

10x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	59.60	60.79	62.13	62.64	64.50	65.85
Median	61.00	62.00	63.00	62.00	66.00	68.00
Standard Deviation	17.81	19.37	19.31	19.66	19.86	20.42
Minimum	10.00	17.00	16.00	10.00	15.00	13.00
Maximum	96.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.56	0.61	0.61	0.62	0.63	0.65

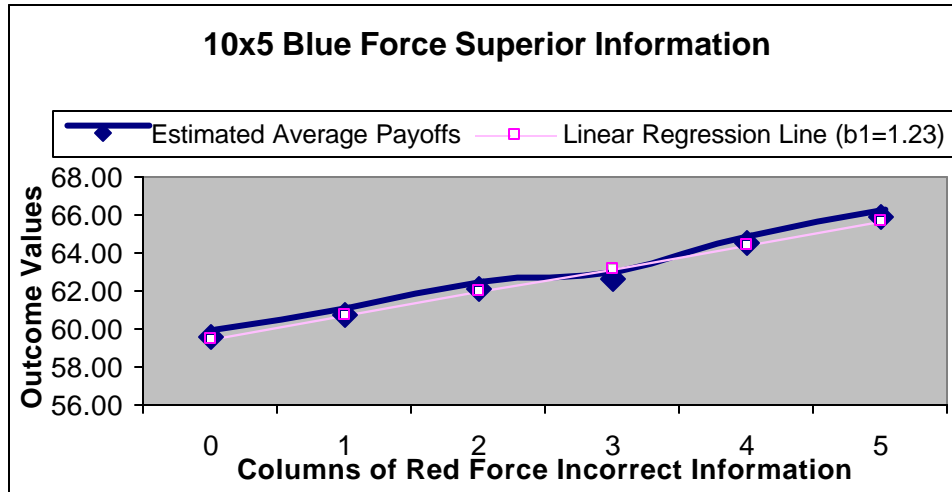


Figure 4.9. Graph of 10×5 Matrix With Red Force Degrading Information.

For the 10×5 payoff matrix, the Blue force is a more capable force than the Red force by possessing twice as many strategy choices with results similar to those observed in the 5×3 case (see Table 4.5). As the quantity of poor choices increase for the Red force from 0 to 5, the Blue force's payoff advantage of 6.25 improves approximately the same as the results of the 5×3 matrix by a value of 7.49, but considerably more than that observed in the 10×3 case for a value of 3.73 (see Table 4.8). These results suggest that since the Blue force has less of an advantage in the 10×5 matrix than in the 10×3, information superiority has more added value in the 10×5 matrix than in the 10×3 case. In Figure 4.9, the payoff outcome values scale is reduced to (56 to 68) since the range of the estimated average payoff is less than 10.

Table 4.10. Data for 10×10 Matrix With Red Force Degrading Information.

10x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	50.01	50.48	50.84	51.40	52.83	54.20	55.90	57.17	57.25	57.34	57.89
Median	50.00	51.00	51.00	50.00	51.00	53.00	56.00	57.00	58.00	57.00	59.00
Standard Deviation	19.51	21.15	20.23	22.05	21.70	22.26	22.78	23.83	23.96	23.76	24.47
Minimum	7.00	0.00	9.00	3.00	7.00	7.00	1.00	6.00	3.00	9.00	1.00
Maximum	93.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.62	0.67	0.64	0.70	0.69	0.70	0.72	0.75	0.76	0.75	0.77

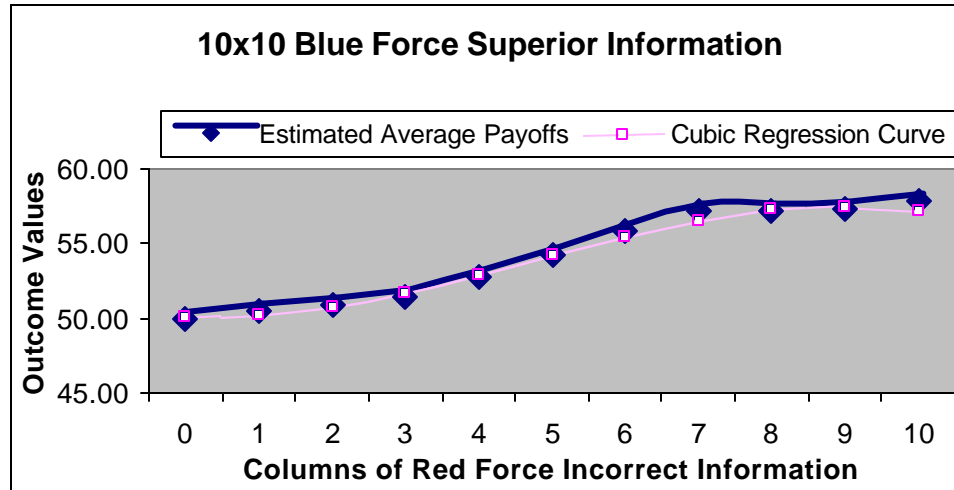


Figure 4.10. Graph of Data for 10×10 Matrix With Red Force Degrading Information.

The 10×10 symmetric payoff matrix, like that of the 3×3 (see Table 4.2) and the 5×5 (see Table 4.6), possesses the same number of strategies for opposing forces, therefore providing no advantages in force capability. As the quantity of incorrect information for the Red force increases from 0% to 100%, the Blue force's payoff advantage in the 10×10 matrix (7.88) is the least among the other two symmetric matrix scenarios (11.31 and 9.69 for the 3×3 and 5×5 matrices respectively). The experiment shows a cubic relationship between the battle payoffs and the Red force's increase in poor strategies since most of the benefits of information superiority occur between 3 and 7 bad column choices by the Red force. In Figure 4.10, the payoff outcome values scale is reduced to (45 to 60) since the range of the estimated average payoff is less than 10.

## C. REPLICATION OF BRACKEN AND DARILEK EXPERIMENT OF COMMON AND CORRECT KNOWLEDGE AND INTELLIGENCE WITH VARYING STRATEGY CHOICES

### 1. Introduction

This experiment considers the effects of opposing forces with common and correct knowledge with one of the opposing forces having intelligence, while varying the amount of strategies available for each force. In their research, Braken and Darilek simulate one force having intelligence by allowing the Blue force prior knowledge of the Red force's strategic intentions. Their research is extended by simulating all combinations of asymmetric matrices (3×5, 3×10, 5×3, 5×10, 10×3 and 10×5), in addition to the symmetric matrices (3×3, 5×5 and 10×10).

### 2. Analysis of Data and Graph

Table 4.11. Data for Blue Force With Intelligence.

Statistics	3x3	3x5	3x10	5x3	5x5	5x10	10x3	10x5	10x10
Mean	57.14	49.60	39.61	70.80	63.75	56.53	83.52	79.18	74.73
Median	57.00	49.00	39.00	72.00	65.00	57.00	85.00	80.00	76.00
Standard Deviation	17.91	16.20	13.88	14.13	14.10	12.55	8.82	8.86	8.97
Minimum	7.00	9.00	4.00	22.00	17.00	14.00	38.00	39.00	38.00
Maximum	98.00	92.00	79.00	98.00	94.00	89.00	98.00	97.00	93.00
Mean Std. Error	0.57	0.51	0.44	0.45	0.45	0.40	0.28	0.28	0.28

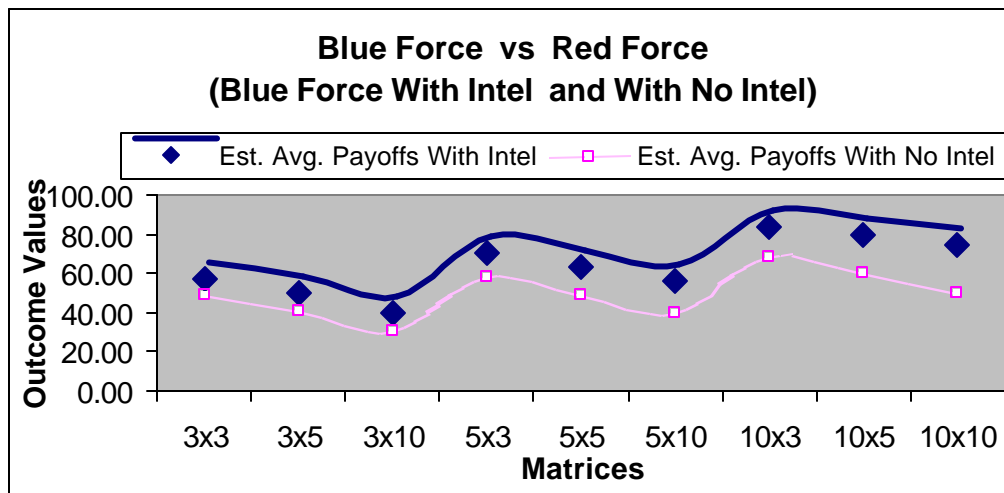


Figure 4.11. Graph of Data for Blue Force With Intelligence.



The presence of intelligence by the Blue force increases the payoff value in its favor in every matrix payoff scenario. The Blue force's advantage due to intelligence in the 10×10 case is 24.57, payoff of 50.16 with no intelligence (see Table 4.1) and 74.73 with intelligence (see Table 4.11), as compared to only an 8.21 advantage in the 3×3 battle scenario. This suggests that the more choices opposing forces have, the more important it is to have knowledge of an opponent's strategic intentions. The highest payoff occurs in the 10×3 matrix at 83.52, whereas the lowest is for the 3×10 case. These results suggest that prior knowledge of the opposing force's decisions provides the greatest benefit to the force with the highest number of options to counter with.

#### **D. VALUE OF VARIOUS LEVELS OF INFORMATION WITH INTELLIGENCE**

##### **1. Introduction**

In this experiment, the Blue force possesses intelligence, thereby knowing the choice of strategy of the Red force, however, the effects of prior knowledge of the opponent's strategic intentions are examined when combined with varying the opponents levels of correct information. This is accomplished by altering the amounts of poor strategies available to the Red force by replacing correct information in the column strategies of the Red force with incorrect information (bad columns) from a separate matrix one column at a time (i.e., from 0% to 100%). This experiment addresses the value of one force gaining information superiority, while also maintaining the advantage of intelligence and knowing the choices of strategy that the force with degrading information will ultimately make.

## 2. Analysis of Data and Graphs

Table 4.12. Data for 3×3 Matrix With Blue Force Intel/Red Force Degrading Info.

3x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	57.62	63.36	69.65	74.57
Median	59.00	65.00	73.00	79.00
Standard Deviation	18.28	20.48	20.08	19.20
Minimum	7.00	4.00	11.00	10.00
Maximum	96.00	99.00	99.00	99.00
Mean Std. Error	0.58	0.65	0.63	0.61

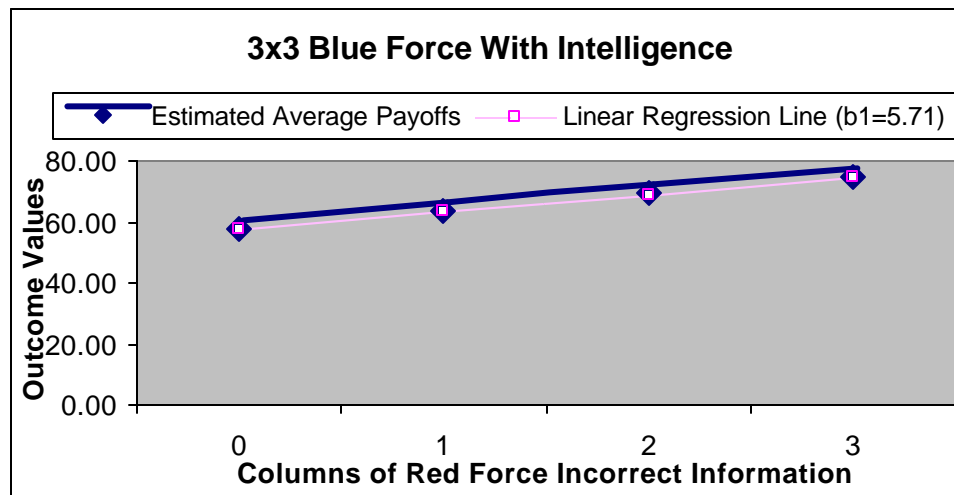


Figure 4.12. Graph of Data for 3×3 Matrix With Blue Force Intel/Red Force Degrading Info.

For the symmetric 3×3 matrix, the payoff value increases in favor of the Blue force, as the quantity of incorrect column strategies for the Red force increases from 0 to 3, by a value of 16.92 with Blue force intelligence (see Table 4.12) and 11.31 without intelligence (see Table 4.2). The standard deviation for the payoff increases from 0 to 1 bad columns by the Red force, and then decreases from 1 to 3. These results suggest that the use of intelligence (i.e., through reconnaissance) allows the Blue force to improve the accuracy of its decision-making. And that by knowing in advance the strategy choices of the Red force, the Blue force can discard less reliable information and focus on its strategies that will maximize the outcome in its favor based on Red force decisions.

Table 4.13. Data for 3×5 Matrix With Blue Force Intel/Red Force Degrading Info.

3x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	50.04	55.57	59.64	64.22	69.09	74.93
Median	51.00	55.00	59.00	65.00	72.00	79.00
Standard Deviation	16.72	20.08	21.77	22.06	21.32	18.91
Minimum	3.00	4.00	7.00	4.00	2.00	14.00
Maximum	92.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.53	0.64	0.69	0.70	0.67	0.60

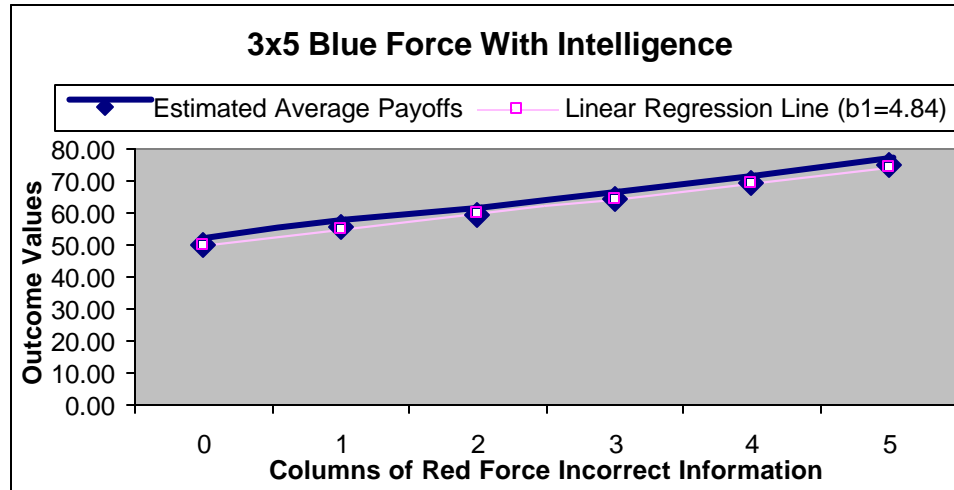


Figure 4.13. Graph of Data for 3×5 Matrix With Blue Force Intel/Red Force Degrading Info.

In the case of the 3×5 scenario, the Red force, with more strategy choices, has an advantage over the Blue force, which is significantly reduced by the Blue force's use of intelligence. Knowing the strategy choices of the Red force, the Blue force is able to improve the battle payoffs in its favor even while having less strategy choices itself. As the quantity of incorrect column strategies for the Red force increases from 0 to 5, the outcome value of the battle increases further in favor of the Blue force by 24.89 with intelligence, as compared to 16.48 without intelligence (see Table 4.3). The average increase in payoff in this scenario is from 41.28 (see Table 4.3) with no Blue force intelligence and both forces having common knowledge to 74.93 with the Blue force possessing both intelligence and information superiority.

Table 4.14. Data for 3×10 Matrix With Blue Force Intel/Red Force Degrading Info.

3x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	40.73	43.16	46.39	51.28	56.08	57.78	60.58	64.54	68.68	70.83	74.37
Median	41.00	42.00	43.00	48.00	54.00	54.00	60.00	66.00	74.00	76.00	78.00
Standard Deviation	14.03	17.98	20.36	22.42	23.63	24.12	23.85	24.26	23.29	21.40	18.78
Minimum	5.00	3.00	3.00	2.00	8.00	3.00	5.00	3.00	4.00	6.00	8.00
Maximum	78.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.44	0.57	0.64	0.71	0.75	0.76	0.75	0.77	0.74	0.68	0.59

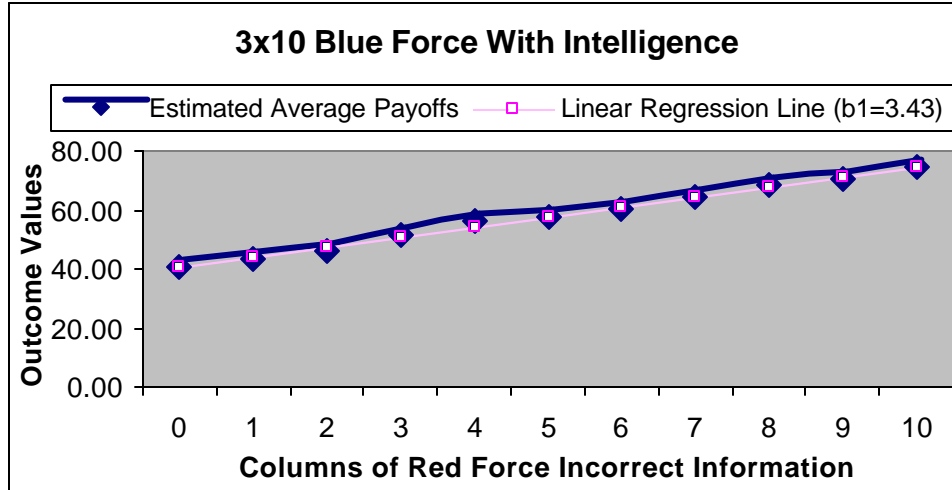


Figure 4.14. Graph of Data for 3×10 Matrix With Blue Force Intel/Red Force Degrading Info.

In the 3×10 matrix, as the number of Red force incorrect column strategies increases from 0 to 10, the battle outcomes increase significantly in favor of the Blue force, by 33.64 with Blue force intelligence, and by 22.36 without the use of intelligence (see Table 4.4). Table 4.4 shows that without intelligence, the payoff favors the Red force until 9 incorrect strategies are reached, whereas with intelligence, the Blue force assumes the advantage after the Red forces obtains only 3 bad choices. The increase in payoff in this scenario is from 30.48 (see Table 4.4) with no Blue force intelligence to 74.37 with the Blue force possessing both intelligence and information superiority. These results suggest that the presence of intelligence allows the Blue force to neutralize the Red force's advantage of having more options and more capable forces at a faster rate.

Table 4.15. Data for 5×3 Matrix With Blue Force Intel/Red Force Degrading Info.

5x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	70.83	74.92	79.53	82.96
Median	72.00	77.00	83.00	87.00
Standard Deviation	14.25	14.79	14.88	14.10
Minimum	14.00	25.00	17.00	24.00
Maximum	98.00	99.00	99.00	99.00
Mean Std. Error	0.45	0.47	0.47	0.45

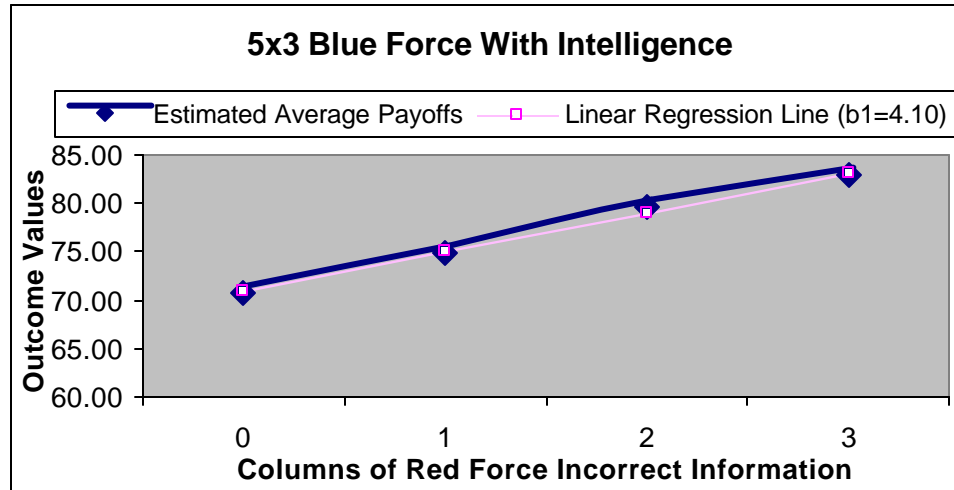


Figure 4.15. Graph of Data for 5×3 Matrix With Blue Force Intel/Red Force Degrading Info.

In the case of the 5×3 matrix, the Blue force possesses a higher number of options and a more capable force. However, when the Red force strategy choices are rendered 100% incorrect and the Blue force gains information superiority, the estimated average payoff value increases from 58.98 (see Figure 4.5) to 82.96. The relationship between outcome values and columns of Red force incorrect information is linear with a constant standard deviation of approximately 14.50. In Figure 4.15, the payoff outcome values scale is reduced to (60 to 85). These results suggest that since the Blue force is superior, intelligence provides little added benefit in advantage when combined with information superiority.

Table 4.16. Data for 5×5 Matrix With Blue Force Intel/Red Force Degrading Info.

5x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	64.14	68.28	71.83	76.23	79.32	82.58
Median	66.00	69.00	73.00	78.00	83.00	86.00
Standard Deviation	14.18	15.95	16.96	16.88	15.88	13.79
Minimum	14.00	17.00	14.00	12.00	19.00	23.00
Maximum	94.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.45	0.50	0.54	0.53	0.50	0.44

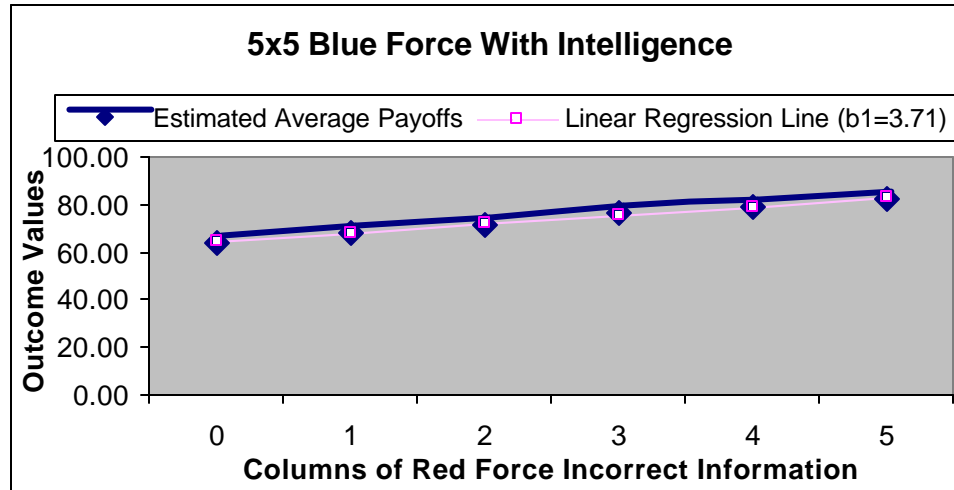


Figure 4.16. Graph of Data for 5×5 Matrix With Blue Force Intel/Red Force Degrading Info.

For the symmetric 5×5 matrix, the battle payoff values increase in favor of the Blue force, as the quantity of incorrect column strategies for the Red force increase from 0 to 5, by a value of 18.44 with Blue force intelligence (see Table 4.16) and 9.66 without intelligence (see Table 4.6). This suggests that the total increase in payoff due to information dominance for this scenario is from 50.49 (see Table 4.6) with no Blue force intelligence and both forces having common knowledge to 82.58 (see Table 4.16) with the Blue force possessing both intelligence and information superiority. The relationship between outcome values and columns of Red force incorrect information is linear, and the standard deviations for the payoffs increase slightly from 0 to 2 bad column choices and decrease from 2 to 5 bad column strategies.

Table 4.17. Data for 5×10 Matrix With Blue Force Intel/Red Force Degrading Info.

5x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	56.98	59.66	61.61	65.01	66.17	71.11	72.54	74.76	77.65	80.49	82.52
Median	58.00	60.00	61.00	64.00	64.00	72.00	74.00	78.00	82.00	85.00	87.00
Standard Deviation	12.46	15.25	16.99	17.32	18.40	18.53	18.85	18.17	17.64	16.14	14.55
Minimum	16.00	8.00	13.00	15.00	8.00	13.00	20.00	15.00	20.00	22.00	28.00
Maximum	85.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.39	0.48	0.54	0.55	0.58	0.59	0.60	0.57	0.56	0.51	0.46

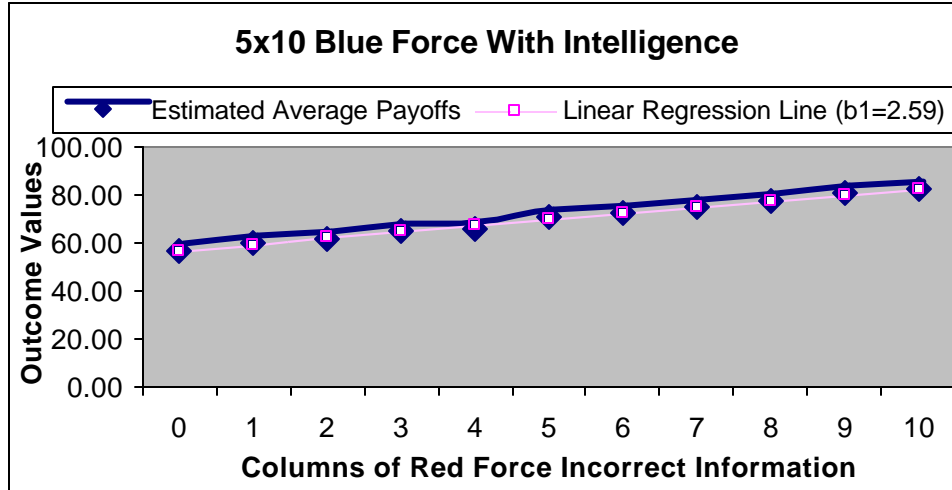


Figure 4.17. Graph of Data for 5×10 Matrix With Blue Force Intel/Red Force Degrading Info.

For the 5×10 matrix, the Blue force without intelligence (see Table 4.7) obtains the advantage after 7 bad Red force strategies, however, with the employment of intelligence, the Blue force has the advantage even at 0 bad Red force choices. The total increase in payoff due to information dominance for this scenario is from 39.70 (see Table 4.7) with no Blue force intelligence and both forces having common knowledge to 82.52 (see Table 4.17) with the Blue force possessing both intelligence and information superiority. These results show the significant benefits of information dominance for a force that may have a substantial disadvantage.

Table 4.18. Data for 10×3 Matrix With Blue Force Intel/Red Force Degrading Info.

10x3 Statistics	0 Bad	1 Bad	2 Bad	3 Bad
Mean	83.02	85.69	88.12	89.79
Median	84.00	87.00	90.00	92.00
Standard Deviation	9.18	9.25	9.21	9.17
Minimum	45.00	42.00	52.00	42.00
Maximum	99.00	99.00	99.00	99.00
Mean Std. Error	0.29	0.29	0.29	0.29

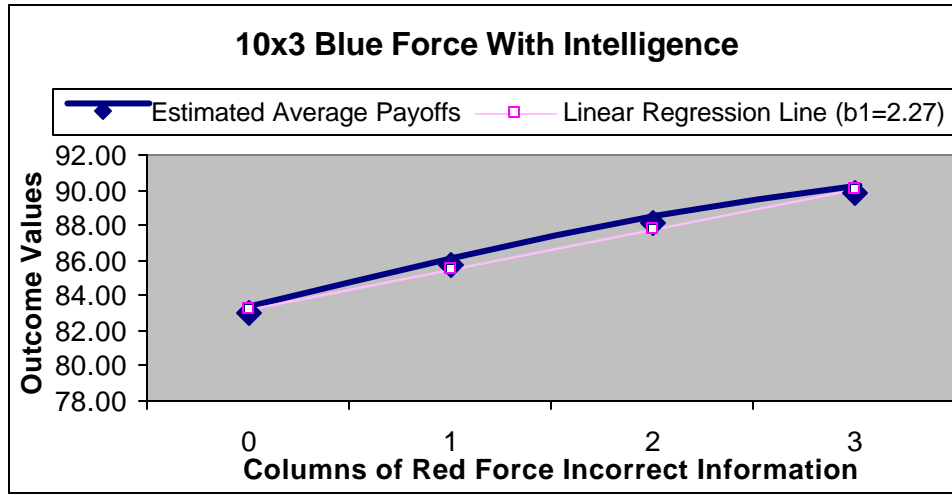


Figure 4.18. Graph of Data for 10×3 Matrix With Blue Force Intel/Red Force Degrading Info.

For the 10×3 matrix, the Blue force possesses an inherent advantage in battle due to its higher number of strategies. The additional employment of Blue force intelligence combined with Red force loss of correct decision-making information results in a Blue force payoff value increase from 83.02 to 89.79 for a payoff gain of only 6.77. Similar to the results in the 5×3 matrix, since the Blue force is superior, intelligence provides little added benefit in advantage when combined with information superiority. In Figure 4.18, the payoff outcome values scale is reduced to (78 to 92).



Table 4.19. Data for 10x5 Matrix With Blue Force Intel/Red Force Degrading Info.

10x5 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad
Mean	79.62	82.09	84.52	86.12	88.45	90.33
Median	81.00	83.00	86.00	88.00	91.00	93.00
Standard Deviation	8.78	10.36	10.33	10.08	9.29	8.30
Minimum	40.00	39.00	41.00	38.00	50.00	50.00
Maximum	97.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.28	0.33	0.33	0.32	0.29	0.26

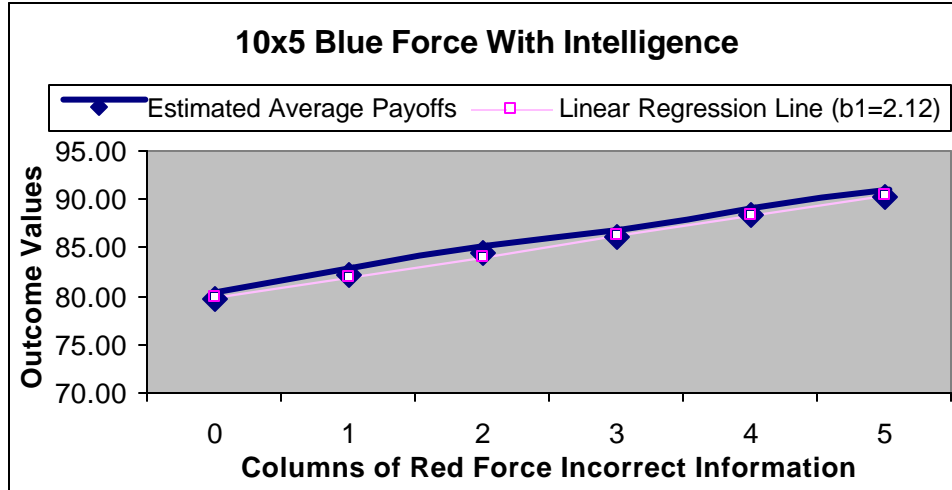


Figure 4.19. Graph of Data for 10x5 Matrix With Blue Force Intel/Red Force Degrading Info.

In the case of the 10x5 payoff matrix, the Blue force possesses less of an advantage over the Red force, as compared to the 10x3 case (see Table 4.18), since the disparity in the amount of force options is smaller. For the 10x5 payoff matrix, the Red force's loss of correct decision-making information results in a greater payoff gain for the Blue force of 10.17, (see Table 4.19), as compared to 6.77 for the 10x3 case (see Table 4.18). In the 10x5 matrix, the relationship between payoffs and increasing superiority is linear. These results support that since the Blue force has less of an advantage in the 10x5 case, the added value in payoff is greater in the 10x5 case than the value in the 10x3 case. In figure 4.19, the payoff outcome values scale is reduced to (70 to 95).

Table 4.20. Data for 10×10 Matrix With Blue Force Intel/Red Force Degrading Info.

10x10 Statistics	0 Bad	1 Bad	2 Bad	3 Bad	4 Bad	5 Bad	6 Bad	7 Bad	8 Bad	9 Bad	10 Bad
Mean	74.96	76.67	78.38	79.61	81.36	82.73	84.33	85.93	87.41	89.18	90.52
Median	76.00	77.00	79.00	80.00	82.00	83.00	86.00	89.00	90.00	92.00	93.00
Standard Deviation	8.55	9.76	10.72	11.33	11.30	11.29	11.39	11.06	10.46	9.46	8.10
Minimum	43.00	32.00	42.00	43.00	34.00	37.00	36.00	48.00	39.00	39.00	51.00
Maximum	94.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00	99.00
Mean Std. Error	0.27	0.31	0.34	0.36	0.36	0.36	0.36	0.35	0.33	0.30	0.26

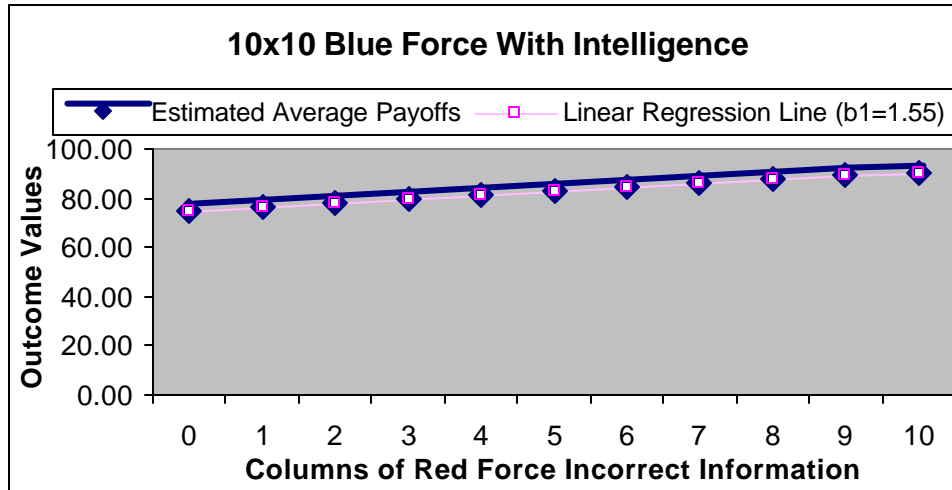


Figure 4.20. Graph of Data for 10×10 Matrix With Blue Force Intel/Red Force Degrading Info.

In the 10×10 matrix scenario, both forces possess their highest levels of capability, and the payoff values favor the Blue force from 74.96 for 0 incorrect Red choices to 90.52 for all 10 incorrect Red force choices. Looking across all cases where the Blue force has its maximum of 10 options (the 10×3, 10×5 and 10×10 matrices), the payoff value is at its highest of approximately 90. This suggests that the higher the number of available choices a force has, the more added benefit the force may receive from intelligence combined with information superiority.

## E. VALUE OF VARYING CAPABILITIES OF FORCES

### 1. Introduction

This experiment addresses the effects of various levels of force capabilities on the outcome values of opposing forces in battle. This scenario alters the boundaries for the random number distributions to represent forces with different levels of capability. For each payoff value in the matrices, the Blue force is simulated as having both superior and

inferior capable forces. Superior forces are simulated by changing the uniform distribution of the Blue force row choices from 50 to 100. Similarly, the Blue force possesses varying levels of inferior capable forces by changing the boundaries of the uniform distribution of the row choices from 0 to 50. The varying levels of superior and inferior capabilities for the Blue force are conducted by changing its row strategies from 0% to 100%, one row at a time.

## 2. Analysis of Data and Graphs

Table 4.21. Data for 3×3 Matrix With Blue Force Superior Capabilities.

3x3 Statistics	0 Superior	1 Superior	2 Superior	3 Superior
Mean	49.80	67.11	71.37	74.15
Median	49.00	66.00	71.00	74.00
Standard Deviation	19.83	10.98	10.30	9.85
Minimum	2.00	50.00	50.00	53.00
Maximum	96.00	97.00	96.00	95.00
Mean Std. Error	0.63	0.35	0.33	0.31

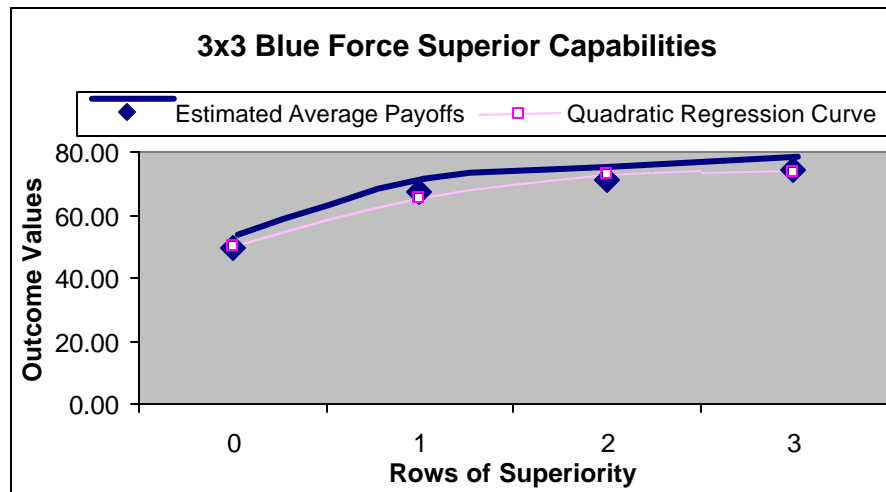


Figure 4.21. Graph of Data for 3×3 Matrix With Blue Force Superior Capabilities.

In this experiment, the 3×3 payoff matrix scenario produces the largest amount of gain in favor of the Blue force when it has 1 superior capability and levels off as the remaining 2 capabilities become superior. The payoff increase for the use of the first superior row is 17.31. The payoff increase for the use of the remaining second and third superior rows is only 7.04. Figure 4.21 shows a quadratic relationship between the estimated average payoffs and the increasing rows of superiority. These results suggest

that even though both forces have the same number of options, the presence of just 1 superior option allows the Blue force to utilize its superior capabilities to create a significant advantage over the opposing Red force.

Table 4.22. Data for 3×3 Matrix With Blue Force Inferior Capabilities.

3x3 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior
Mean	49.25	41.36	32.97	25.18
Median	48.00	40.00	31.00	25.00
Standard Deviation	19.44	18.47	15.27	9.96
Minimum	3.00	4.00	2.00	2.00
Maximum	96.00	94.00	89.00	48.00
Mean Std. Error	0.61	0.58	0.48	0.31

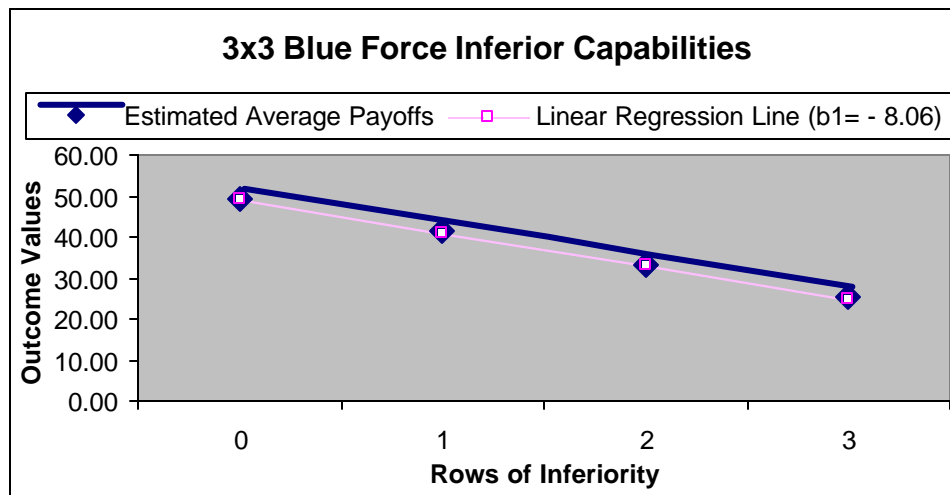


Figure 4.22. Graph of Data for 3×3 Matrix With Blue Force Inferior Capabilities.

For the 3×3 symmetric matrix, as the Blue force capabilities become increasingly inferior to those of the Red force, the outcome values decrease linearly in favor of the Red force from 49.25 to 25.18 for a total loss of 24.07. The battle payoff standard deviation also decreases from 19.44 to 9.96, as the Blue force's inferiority increases. These results suggest that as the Blue force's choices become inferior, the advantage gained by the Red force is equally distributed between each inferior option.

Table 4.23. Data for 3×5 Matrix With Blue Force Superior Capabilities.

3x5 Statistics	0 Superior	1 Superior	2 Superior	3 Superior
Mean	40.35	62.50	66.73	69.96
Median	40.00	61.00	66.00	70.00
Standard Deviation	17.65	8.92	9.04	8.63
Minimum	3.00	50.00	50.00	51.00
Maximum	89.00	92.00	96.00	94.00
Mean Std. Error	0.56	0.28	0.29	0.27

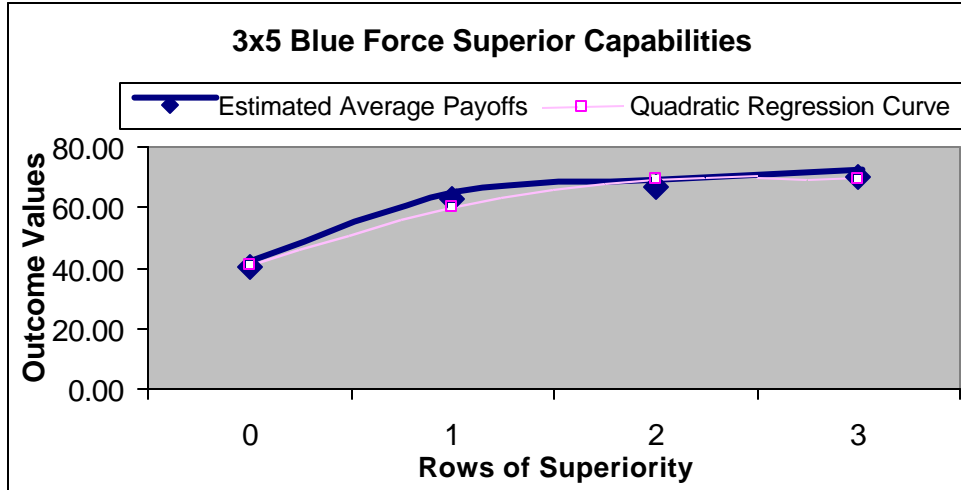


Figure 4.23. Graph of Data for 3×5 Matrix With Blue Force Superior Capabilities.

In the case of the 3×5 matrix, where the Red force possesses slightly more strategic options and an initial force advantage of 40.35, the presence of the first superior capable Blue force provides the most benefit, as it produces an increase in payoff of 22.15. The increase in superiority of the remaining 2 Blue force options increases the Blue force's advantage by only a value of 7.40. Similar to the 3×3 case, the payoff increase has a quadratic relationship with the rows of superiority, and the results show that the presence of good information allows the Blue force to use its superior capabilities.

Table 4.24. Data for 3×5 Matrix With Blue Force Inferior Capabilities.

3x5 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior
Mean	41.34	33.19	25.80	19.97
Median	42.00	32.00	25.00	20.00
Standard Deviation	18.05	16.35	12.33	9.06
Minimum	2.00	4.00	1.00	0.00
Maximum	89.00	84.00	75.00	43.00
Mean Std. Error	0.57	0.52	0.39	0.29

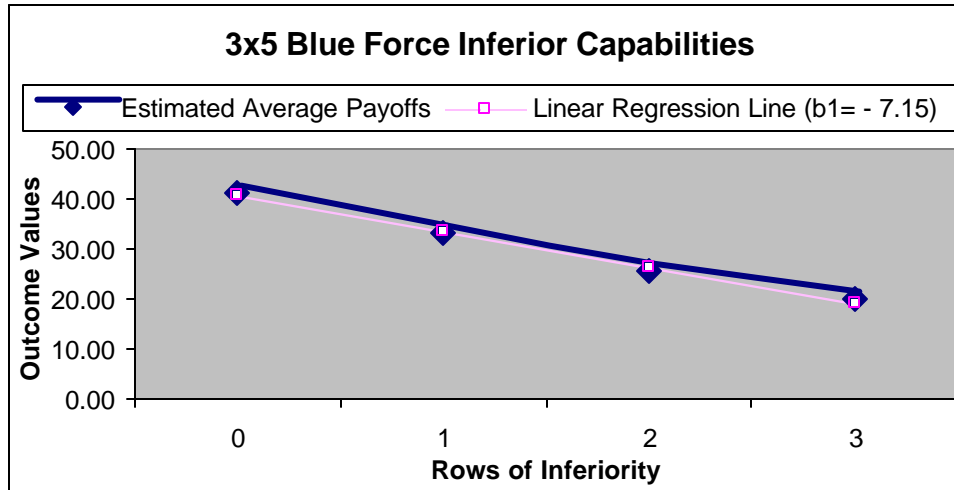


Figure 4.24. Graph of Data for 3×5 Matrix With Blue Force Inferior Capabilities.

For the 3×5 payoff matrix, as the Blue force chooses between increasingly inferior capable force strategies, the total payoff decreases linearly for a total loss of 21.37. The payoff loss for the Blue force in the 3×5 matrix is less than that for the 3×3 matrix which has a payoff loss of 24.07 (see Table 4.22). From this data, we see that if a force has a disadvantage, the loss of effectiveness of these force capabilities reduces the payoff by a comparatively smaller margin than the loss in the symmetric 3×3 case.

Table 4.25. Data for 3×10 Matrix With Blue Force Superior Capabilities.

3x10 Statistics	0 Superior	1 Superior	2 Superior	3 Superior
Mean	30.33	58.52	62.19	65.08
Median	29.00	57.00	61.00	64.00
Standard Deviation	15.29	6.87	7.18	7.87
Minimum	1.00	50.00	50.00	50.00
Maximum	75.00	84.00	86.00	95.00
Mean Std. Error	0.48	0.22	0.23	0.25

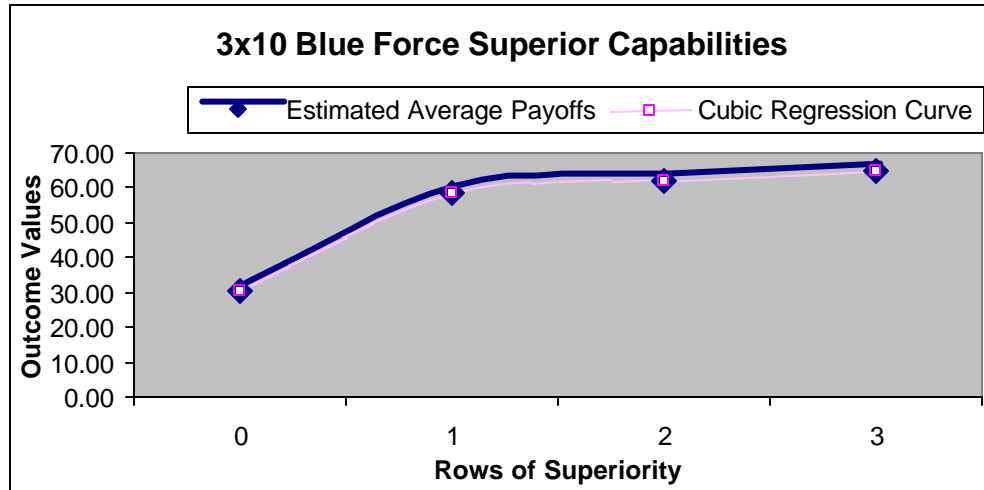


Figure 4.25. Graph of Data for 3×10 Matrix With Blue Force Superior Capabilities.

For the 3×10 payoff matrix, the Red force initially has an even greater force advantage (30.33), as compared to 41.34 for the 3×5 case (see Table 4.24). However, the battle outcome increases substantially in favor of the Blue force, after the addition of just 1 superior capable force strategy, by a value 28.19. The increase in payoff for the remaining second and third superior capable Blue force strategies is only 6.56. These results show an even more dominant effect from the presence of the first good option, as the increase in payoff displays a cubic relationship with the rows of superiority for the Blue force. The Blue force is, therefore, able to overcome a severe disadvantage if it knows its best capable options.

Table 4.26. Data for 3×10 Matrix With Blue Force Inferior Capabilities.

3x10 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior
Mean	30.73	24.96	18.92	15.37
Median	30.00	24.00	18.00	14.00
Standard Deviation	15.38	12.85	9.81	7.95
Minimum	0.00	1.00	0.00	1.00
Maximum	80.00	71.00	49.00	40.00
Mean Std. Error	0.49	0.41	0.31	0.25

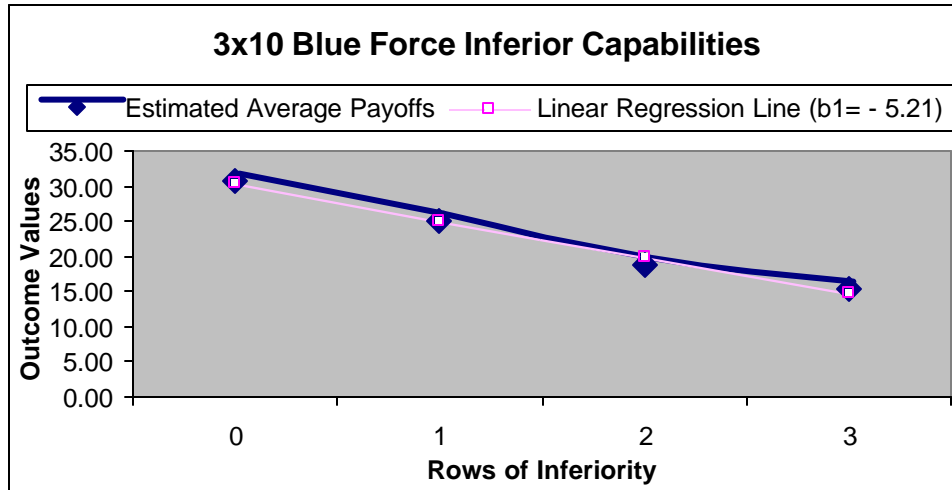


Figure 4.26. Graph of Data for 3×10 Matrix With Blue Force Inferior Capabilities.

In the case of the 3×10 matrix, where the Red force possesses significantly more strategies and thus has an inherent advantage over the Blue force, the reduction in force capability (from 0 inferior to all 3 inferior) by the Blue force, serves to further benefit the Red force with the lowest simulated outcome value of 15.37. This decrease in payoff value is also reflected in the value of the average minimum outcome value of 0.50. These results suggest that from Red's perspective, bad rows increase the advantage linearly in their favor.



Table 4.27. Data for 5×3 Matrix With Blue Force Superior Capabilities.

5x3 Statistics	0 Superior	1 Superior	2 Superior	3 Superior	4 Superior	5 Superior
Mean	58.72	70.69	72.89	75.76	77.57	78.81
Median	59.00	71.00	73.00	76.00	78.00	79.00
Standard Deviation	18.27	11.41	10.50	10.09	9.41	8.85
Minimum	9.00	50.00	50.00	53.00	54.00	56.00
Maximum	97.00	97.00	98.00	97.00	97.00	97.00
Mean Std. Error	0.58	0.36	0.33	0.32	0.30	0.28

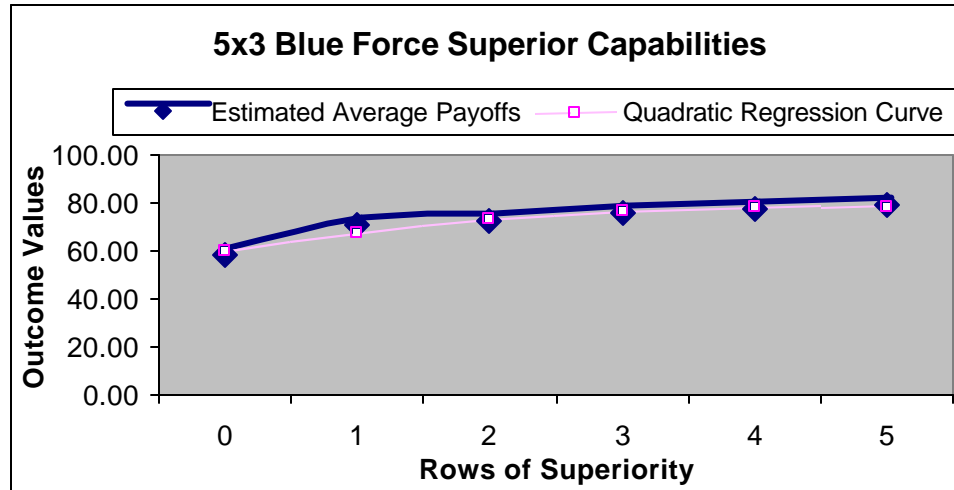


Figure 4.27. Graph of Data for 5×3 Matrix With Blue Force Superior Capabilities.

For the 5×3 matrix, the inherent advantage of the Blue force with more available options is improved by the addition of forces with superior capabilities. The majority of the payoff increase occurs after the use of the first superior choice, which produces a gain of 11.97 and a near quadratic relationship. For the remaining 4 superior capable courses of action, the increase is only 8.12 and displays a piece-wise linear relationship. These results suggest that a force's advantage after gaining one good option is less when it is the superior force.

Table 4.28. Data for 5×3 Matrix With Blue Force Inferior Capabilities.

5x3 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior
Mean	58.54	53.63	50.65	42.64	36.00	29.15
Median	59.00	53.00	49.00	41.00	36.00	29.00
Standard Deviation	17.75	18.13	18.81	16.61	13.20	9.17
Minimum	8.00	5.00	10.00	6.00	6.00	3.00
Maximum	96.00	95.00	97.00	92.00	90.00	49.00
Mean Std. Error	0.56	0.57	0.59	0.53	0.42	0.29

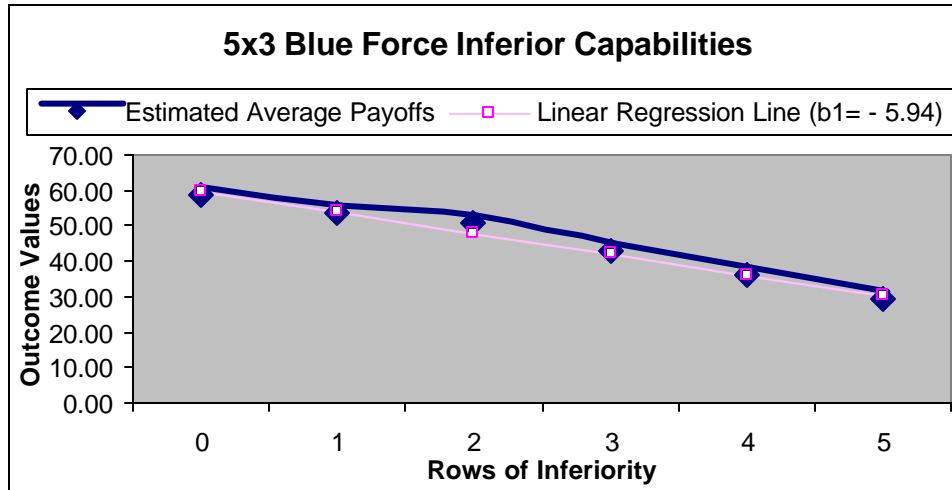


Figure 4.28. Graph of Data for 5×3 Matrix With Blue Force Inferior Capabilities.

For the 5×3 matrix, the Blue force's advantage of having two more strategic options is reduced to a value of 50.65 for 2 of 5 inferior choices and is further reduced in favor of the Red force when 5 of 5 choices are inferior to a value of 29.15. From these results, we see that a force with a substantial advantage may protect itself by reducing the rate of payoff loss to the extent of having 2 inferior capabilities before its opponent may gain the advantage.

Table 4.29. Data for 5×5 Matrix With Blue Force Superior Capabilities.

5x5 Statistics	0 Superior	1 Superior	2 Superior	3 Superior	4 Superior	5 Superior
Mean	50.20	65.73	68.85	71.79	72.98	74.51
Median	50.00	65.00	68.00	72.00	73.00	74.00
Standard Deviation	18.50	10.40	9.64	9.56	9.22	9.43
Minimum	8.00	50.00	50.00	50.00	52.00	54.00
Maximum	94.00	96.00	93.00	97.00	96.00	96.00
Mean Std. Error	0.58	0.33	0.30	0.30	0.29	0.30

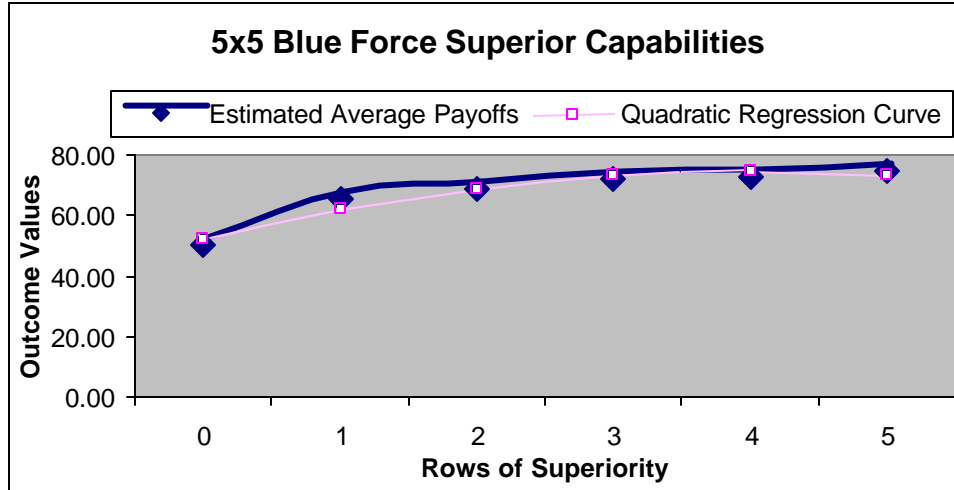


Figure 4.29. Graph of Data for 5×5 Matrix With Blue Force Superior Capabilities.

For the 5×5 symmetric scenario, since neither side benefits by having an advantage in the number of strategies, the increase in the battle outcome is due only to the superiority of the Blue force options. The majority of the Blue force payoff gain occurs after the addition of just 1 superior strategic option for an increase of 15.53. The increase for the remaining 4 superior capable Blue force row options is only 8.78. A quadratic or perhaps piece-wise linear relationship exists between the estimated average payoffs and the rows of Blue force superiority. The results show that one good choice allows the Blue force to use its best choice and increase the payoff significantly in its favor.

Table 4.30. Data for 5×5 Matrix With Blue Force Inferior Capabilities.

5x5 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior
Mean	49.61	45.10	41.39	34.34	29.03	24.80
Median	50.00	44.00	40.00	33.00	29.00	25.00
Standard Deviation	17.97	17.99	16.80	14.60	11.41	9.33
Minimum	6.00	4.00	3.00	4.00	5.00	2.00
Maximum	93.00	87.00	91.00	94.00	81.00	48.00
Mean Std. Error	0.57	0.57	0.53	0.46	0.36	0.29

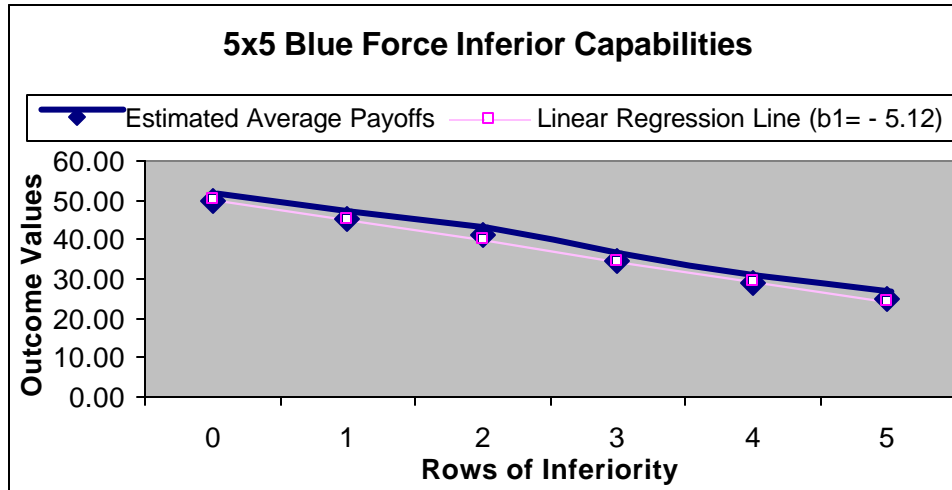


Figure 4.30. Graph of Data for 5×5 Matrix With Blue Force Inferior Capabilities.

The results of the 5×5 symmetric payoff matrix are similar to those of the 3×3 matrix (see Table 4.22). As the Blue force capabilities become increasingly inferior to those of the Red force, the outcome values decrease linearly from 49.61 to 24.80 for a total loss of 24.81 in favor of the Red force, as compared to a loss of 24.07 for the 3×3 matrix (see Table 4.22). The battle payoff standard deviation also decreases from 17.97 to 9.33 (from 19.44 to 9.96 for the 3×3 matrix) as the Blue force's inferiority increases.

Table 4.31. Data for 5×10 Matrix With Blue Force Superior Capabilities.

5x10 Statistics	0 Superior	1 Superior	2 Superior	3 Superior	4 Superior	5 Superior
Mean	38.74	61.75	64.91	67.15	68.40	69.82
Median	38.00	61.00	64.00	67.00	68.00	69.00
Standard Deviation	17.69	8.21	8.29	8.58	8.77	8.86
Minimum	3.00	50.00	50.00	50.00	51.00	51.00
Maximum	86.00	89.00	88.00	93.00	89.00	90.00
Mean Std. Error	0.56	0.26	0.26	0.27	0.28	0.28

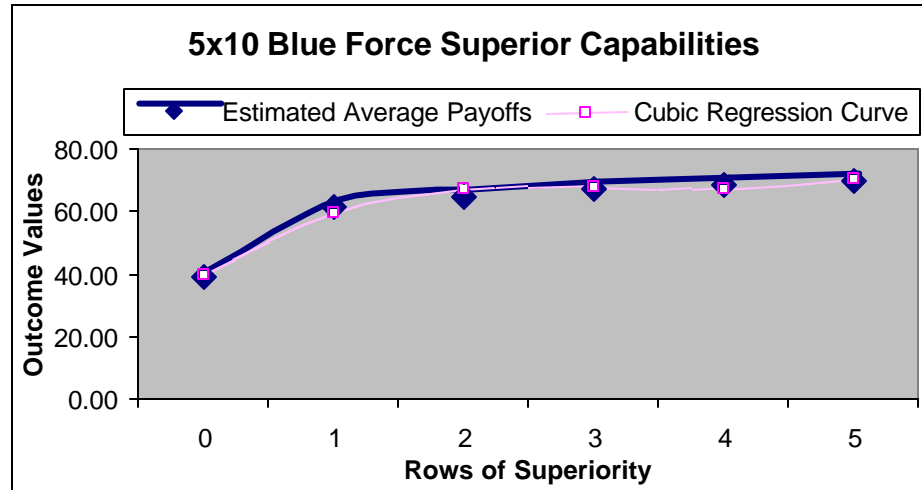


Figure 4.31. Graph of Data for 5×10 Matrix With Blue Force Superior Capabilities.

For the 5×10 matrix, where the Red force initially benefits by having twice as many courses of action as the Blue force, the Blue force neutralizes its disadvantage and changes the outcome payoff in its favor by the largest margin using only 1 superior capable option. The increase in payoff for 1 superior capable Blue force option is 23.01, as compared to an increase of only 8.07 for the remaining 4 superior capable Blue force options. This increase in payoffs displays a cubic or piece-wise linear relationship with the rows of superiority for the Blue force. The results of the 5×10 matrix are similar to both the 3×5 and 3×10 matrices in that the use of just one superior capable option is enough to allow the Blue force to overcome a very severe disadvantage.

Table 4.32. Data for 5×10 Matrix With Blue Force Inferior Capabilities.

5x10 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior
Mean	40.93	35.59	31.65	26.79	22.09	19.41
Median	41.00	34.00	30.00	26.00	21.00	19.00
Standard Deviation	17.76	16.71	15.05	12.64	10.25	8.71
Minimum	5.00	2.00	2.00	3.00	2.00	1.00
Maximum	84.00	81.00	81.00	73.00	62.00	40.00
Mean Std. Error	0.56	0.53	0.48	0.40	0.32	0.28

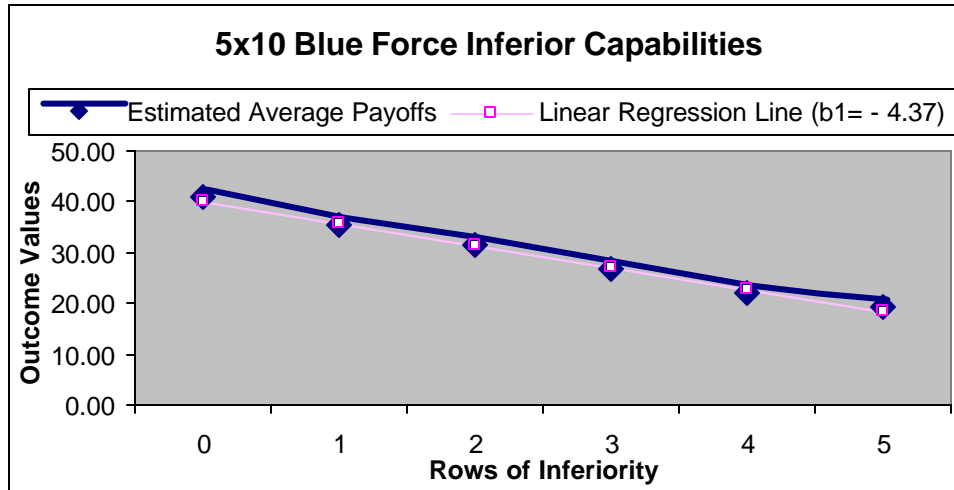


Figure 4.32. Graph of Data for 5×10 Matrix With Blue Force Inferior Capabilities.

In this experiment, the results for the 5×10 payoff matrix are similar to the results of both the 3×5 scenario (see Table 4.24) and the 3×10 scenario (see Table 4.26), where the payoff advantage is initially in favor of the Red force since in each case it has more options than the Blue force. As the Blue force row options increase in inferiority from 0% to 100%, the total Blue force payoff loss is 21.52 for the 5×10 case, as compared to a loss of 21.37 for the 3×5 matrix and 15.36 for the 3×10 matrix.

Table 4.33. Data for 10×3 Matrix With Blue Force Superior Capabilities.

10x3 Statistics	0	1	2	3	4	5	6	7	8	9	10
Superior	Superior	Superior	Superior	Superior	Superior	Superior	Superior	Superior	Superior	Superior	Superior
Mean	68.21	74.57	77.15	78.27	79.65	80.17	81.70	82.25	83.28	82.93	83.72
Median	70.00	75.00	78.00	79.00	80.00	81.00	82.00	83.00	84.00	83.00	84.00
Standard Deviation	15.70	11.71	10.50	9.76	9.32	9.12	8.74	8.58	7.99	8.16	7.64
Minimum	20.00	50.00	50.00	52.00	55.00	55.00	56.00	57.00	59.00	59.00	61.00
Maximum	99.00	99.00	98.00	98.00	98.00	98.00	99.00	98.00	99.00	99.00	98.00
Mean Std. Error	0.50	0.37	0.33	0.31	0.29	0.29	0.28	0.27	0.25	0.26	0.24

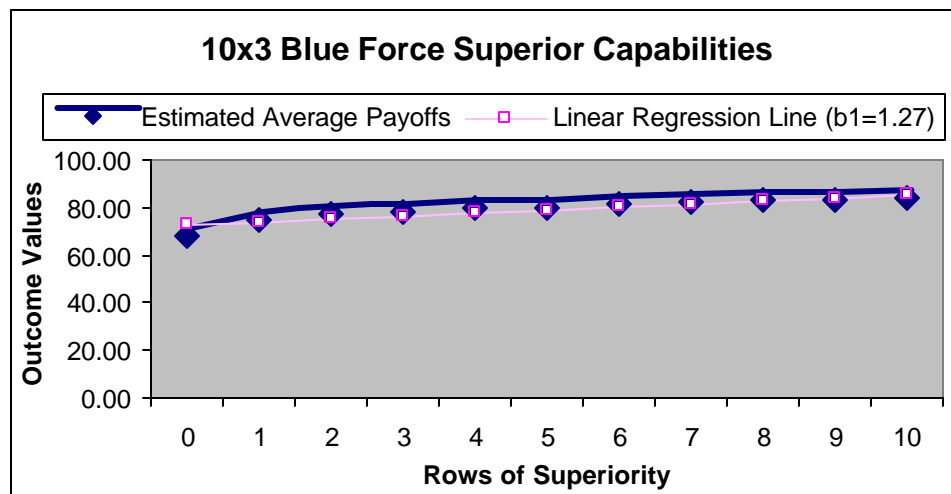


Figure 4.33. Graph of Data for 10×3 Matrix With Blue Force Superior Capabilities.

In the 10×3 scenario, since the Blue force has the advantage, the addition of the Blue force superior row options have a relatively smaller affect on the increase in total payoff. The relationship between payoffs and increasing superiority is linear. In this case, the Blue force possesses 10 available options and receives a comparatively small gain in payoff after the first superior capable choice is included, for a gain of only 6.36. The total payoff after the remaining 9 superior capable Blue force row options are included is only 9.15. These results suggest that for a force with a significant advantage, the effects of superior force capabilities on the payoff outcome are minimal.

Table 4.34. Data for 10×3 Matrix With Blue Force Inferior Capabilities.

10x3 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior	6 Inferior	7 Inferior	8 Inferior	9 Inferior	10 Inferior
Mean	69.14	67.47	65.46	63.58	61.03	58.60	55.48	50.63	45.23	38.87	34.07
Median	70.00	69.00	67.00	64.00	62.00	58.00	55.00	48.00	42.00	38.00	34.00
Standard Deviation	15.67	15.98	16.67	17.21	17.07	16.83	17.70	16.91	16.20	11.65	7.80
Minimum	19.00	25.00	15.00	9.00	20.00	18.00	17.00	15.00	9.00	14.00	8.00
Maximum	98.00	97.00	98.00	98.00	98.00	97.00	96.00	97.00	92.00	85.00	49.00
Mean Std. Error	0.50	0.51	0.53	0.54	0.54	0.53	0.56	0.53	0.51	0.37	0.25

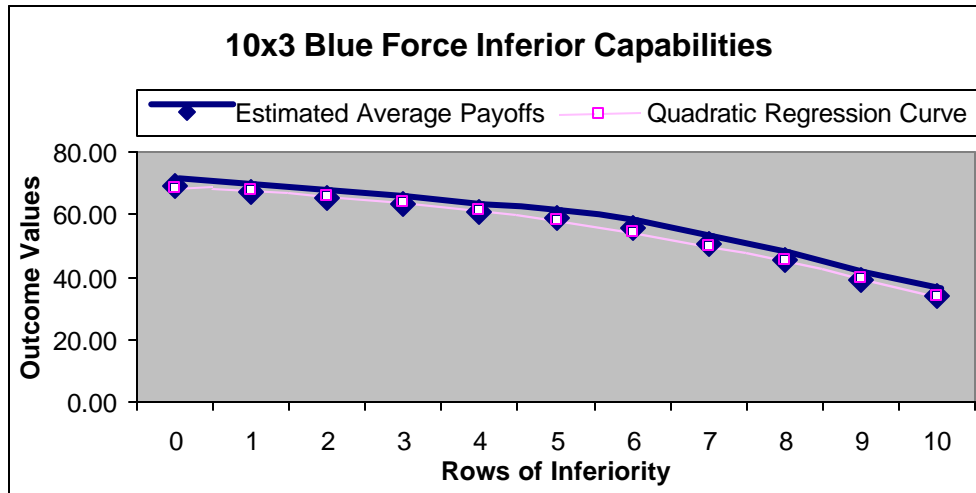


Figure 4.34. Graph of Data for 10×3 Matrix With Blue Force Inferior Capabilities.

In this scenario, the Blue force begins with a significant advantage over the Red force by having more than twice the number of available options. However, as the capabilities of the Blue force row strategies begin to decrease, the estimated average payoff values show a quadratic relationship. The advantage of the Blue force and the high number of available choices, prevent the Blue force from losing its advantage until the 8<sup>th</sup> inferior row choice is reached. These results suggest that a force with a significant advantage is more capable of defending against and preventing the loss in payoff due to a decrease in force capability. The greater the amount of choices, the more protection a force may gain against inferior options.



Table 4.35. Data for 10×5 Matrix With Blue Force Superior Capabilities.

10x5 Statistics	0 Superior	1 Superior	2 Superior	3 Superior	4 Superior	5 Superior	6 Superior	7 Superior	8 Superior	9 Superior	10 Superior
Mean	59.89	69.99	72.04	73.99	74.95	75.77	77.62	78.22	78.47	78.97	79.53
Median	61.00	70.00	73.00	74.00	75.00	76.00	78.00	79.00	79.00	79.00	80.00
Standard Deviation	17.53	11.71	10.99	10.48	9.96	9.43	9.25	9.27	9.26	9.27	8.89
Minimum	15.00	50.00	50.00	51.00	53.00	53.00	54.00	57.00	54.00	58.00	55.00
Maximum	97.00	96.00	96.00	97.00	97.00	97.00	97.00	97.00	97.00	97.00	98.00
Mean Std. Error	0.55	0.37	0.35	0.33	0.31	0.30	0.29	0.29	0.29	0.29	0.28

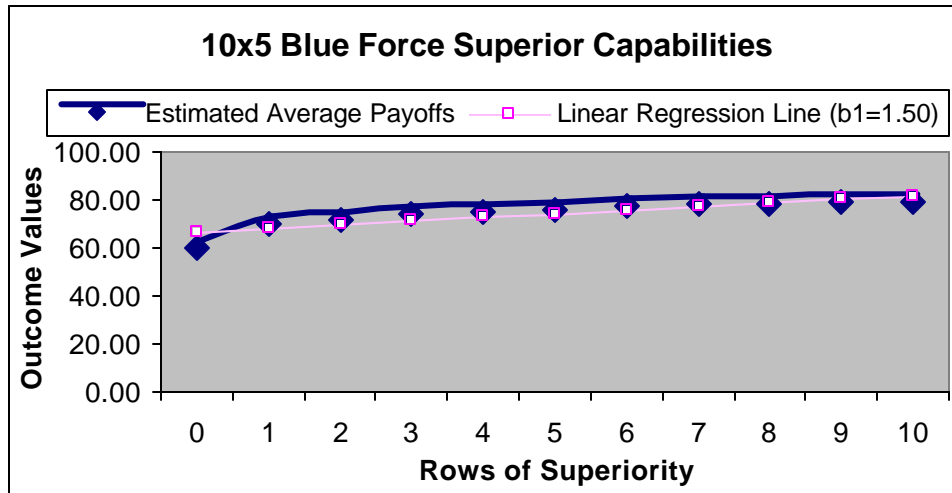


Figure 4.35. Graph of Data for 10×5 Matrix With Blue Force Superior Capabilities.

In the 10×5 payoff matrix, the Blue force has an initial advantage (58.89) over the Red force by having twice as many available options. As a result, the gain in payoff as the Blue force receives superior row options is minimal. For example, the payoff gain provided by the addition of 1 superior row capability is only 10.10 and the payoff gain for the remaining 9 superior row capabilities is even less at 9.54. The relationship between payoffs and increasing superiority is linear or piece-wise linear after the change point of the first superior row option. Similar to the 10×3 and 5×3 matrices, these results suggest that the Blue force's benefit of one good option is comparatively less when it is the superior force.

Table 4.36. Data for 10×5 Matrix With Blue Force Inferior Capabilities.

10x5 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior	6 Inferior	7 Inferior	8 Inferior	9 Inferior	10 Inferior
Mean	59.21	58.47	56.34	54.13	52.41	48.87	45.66	42.42	37.23	32.04	29.08
Median	60.00	59.00	57.00	54.00	52.00	47.00	44.00	40.00	37.00	32.00	29.00
Standard Deviation	17.59	17.72	18.15	17.57	17.79	17.37	16.65	15.87	13.23	10.20	8.64
Minimum	17.00	11.00	13.00	12.00	10.00	11.00	7.00	10.00	9.00	6.00	7.00
Maximum	96.00	96.00	96.00	95.00	94.00	93.00	91.00	90.00	88.00	72.00	48.00
Mean Std. Error	0.56	0.56	0.57	0.56	0.56	0.55	0.53	0.50	0.42	0.32	0.27

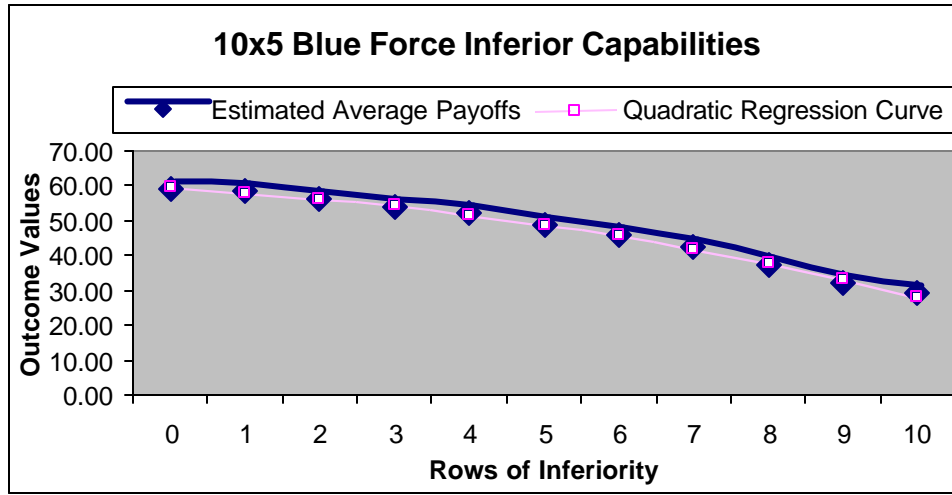


Figure 4.36. Graph of Data for 10×5 Matrix With Blue Force Inferior Capabilities.

In the 10×3 payoff matrix (see Table 4.34), the Red force assumes the advantage as the 8<sup>th</sup> Blue force row choice becomes inferior. In the case of the 10×5 scenario, the payoff value drops off at a faster rate, and the Red force assumes the advantage after only the 5<sup>th</sup> Blue force row choice becomes inferior. As the capabilities of the Blue force row strategies begin to decrease, the estimated average payoff values show a quadratic relationship. These results show that in the 10×5 case, since the Blue force possesses less of an advantage than that of the 10×3, the Blue force becomes more susceptible to inferiority and, therefore, loses the advantage sooner than in the 10×3 case.

Table 4.37. Data for 10×10 Matrix With Blue Force Superior Capabilities.

10x10 Statistics	0 Superior	1 Superior	2 Superior	3 Superior	4 Superior	5 Superior	6 Superior	7 Superior	8 Superior	9 Superior	10 Superior
Mean	50.24	66.23	68.03	70.30	70.87	72.03	72.06	73.24	73.60	74.50	74.53
Median	50.00	65.00	67.00	70.00	71.00	72.00	72.00	74.00	74.00	75.00	74.00
Standard Deviation	18.85	10.30	9.90	10.12	10.15	9.53	9.71	9.83	9.71	9.71	9.72
Minimum	0.00	50.00	50.00	51.00	51.00	52.00	52.00	53.00	54.00	55.00	54.00
Maximum	89.00	92.00	92.00	93.00	91.00	98.00	93.00	96.00	95.00	94.00	96.00
Mean Std. Error	0.60	0.33	0.31	0.32	0.32	0.30	0.31	0.31	0.31	0.31	0.31

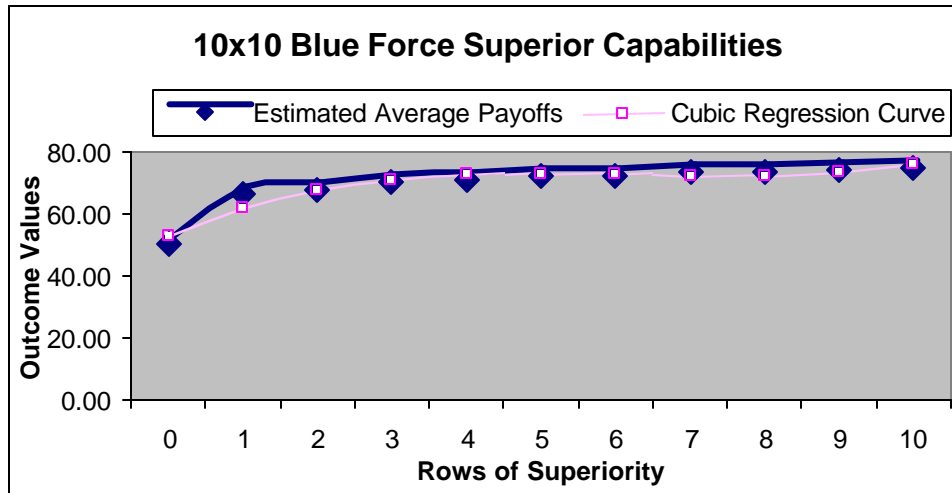


Figure 4.37. Graph of Data for 10×10 Matrix With Blue Force Superior Capabilities.

In the 10×10 symmetric payoff matrix, neither force has an advantage in the number of courses of actions. As the Blue force options increase in superiority from 0% to 100%, there is a big first step and a cubic relationship with the payoff values. The gain at 1 superior option has a value of 15.99. The increase for the remaining 9 superior capable options is only 8.30 and displays a flat, piece-wise linear relationship with the payoff values. Therefore, as the number of available choices increases, the less of an influence each additional superior capability has after the addition of the first superior option. Looking across all of the cases of superior Blue forces, a clear pattern emerges. A piece-wise linear relationship exists with a change point at 1 superior row. Therefore, one good option is all that the Blue force needs, as long as they are aware of it.

Table 4.38. Data for 10×10 Matrix With Blue Force Inferior Capabilities.

10x10 Statistics	0 Inferior	1 Inferior	2 Inferior	3 Inferior	4 Inferior	5 Inferior	6 Inferior	7 Inferior	8 Inferior	9 Inferior	10 Inferior
Mean	49.83	48.24	46.29	44.15	40.94	39.66	35.53	32.00	29.12	26.56	24.21
Median	50.00	48.00	46.00	44.00	40.00	39.00	34.00	32.00	29.00	26.00	24.00
Standard Deviation	19.47	18.94	19.02	18.70	17.55	17.26	14.96	13.73	11.63	10.13	10.01
Minimum	5.00	6.00	7.00	8.00	5.00	4.00	6.00	6.00	4.00	6.00	4.00
Maximum	92.00	90.00	86.00	85.00	87.00	84.00	78.00	84.00	68.00	53.00	46.00
Mean Std. Error	0.62	0.60	0.60	0.59	0.55	0.55	0.47	0.43	0.37	0.32	0.32

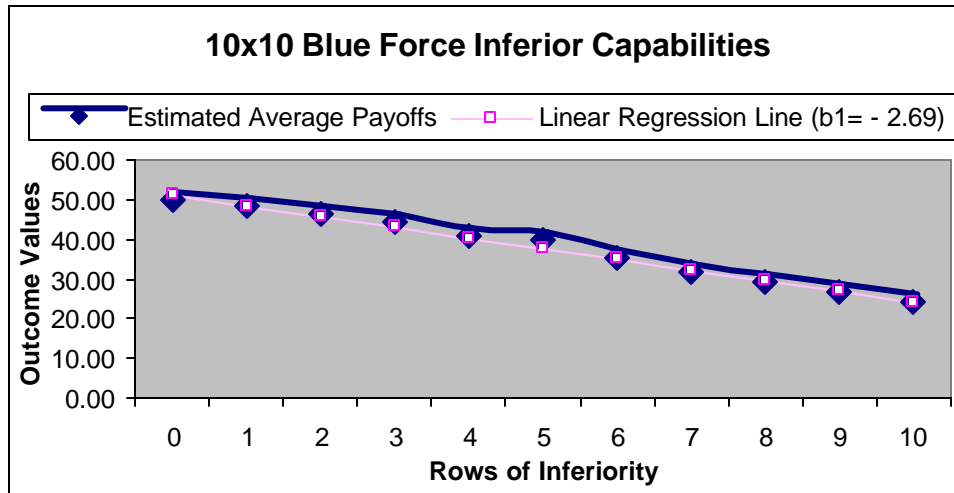


Figure 4.38. Graph of Data for 10×10 Matrix With Blue Force Inferior Capabilities.

The 10×10 payoff matrix results resemble those of both the 3×3 and 5×5 symmetric cases. In each of the three scenarios, each opposing force possesses equal numbers of force capabilities, therefore, neither side has an advantage. As the quality of all of the Blue force's capabilities are reduced to being inferior to the Red force, the value of the game depreciates from 49.83 to 24.21, as compared to 49.25 to 25.18 for the 3×3 matrix (see Table 4.22) and 49.61 to 24.80 for the 5×5 case (see Table 4.30). Looking across all of the scenarios, we see the emerging pattern that as the number of inferior capable rows increases, the payoff advantage increases linearly in favor of the Red force.

## F. VALUE OF VARYING LEVELS OF INFORMATION USING NORMAL DISTRIBUTION PAYOFFS

### 1. Introduction

This experiment extends the methodology used by Bracken and Darilek by generating payoffs for each matrix using the truncated normal distribution with a mean of

50, a standard deviation of  $\sqrt{100^2/12} = 28.87$ , a minimum of 0 and a maximum of 100.

## 2. Analysis of Data and Graph

Table 4.39. Data for Both Forces Using Normal Distribution.

Statistics	3x3	3x5	3x10	5x3	5x5	5x10	10x3	10x5	10x10
Mean	49.99	42.37	35.18	56.46	49.62	41.81	63.77	56.74	49.73
Median	51.00	42.00	35.00	56.00	49.00	41.00	64.00	56.00	50.00
Standard Deviation	14.38	13.14	12.60	13.68	13.33	13.40	12.34	13.51	13.82
Minimum	9.00	10.00	2.00	13.00	13.00	11.00	28.00	24.00	13.00
Maximum	95.00	78.00	69.00	94.00	84.00	93.00	94.00	92.00	86.00
Mean Std. Error	0.45	0.42	0.40	0.43	0.42	0.42	0.39	0.43	0.44

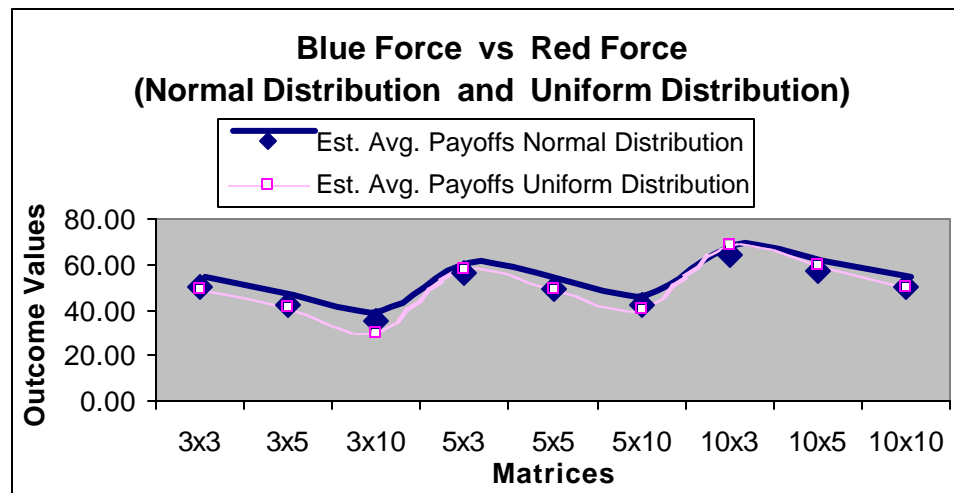


Figure 4.39. Graph of Data for Both Forces Using Normal Distribution.

The results of the data simulating opposing forces in battle with common and correct information using the normal distribution are quite similar to the outcomes for the uniform distribution from 0 to 100 (see Table 4.1). These estimated average payoffs are especially close in value for the symmetric matrices. For the 3×3 payoff matrix, the value for the normal distribution is 49.99 (48.93 for uniform distribution). In the case of the 5×5 matrix, the value for the normal distribution is 49.62 (49.02 for the uniform distribution). And for the 10×10 matrix, the value for the normal distribution is 49.73 (50.16 for the uniform distribution).

The estimated average payoff values for the normal distribution are greater than those of the uniform distribution in the matrix scenarios where the Red force has the higher number of options and the more capable forces. For example, for the  $3 \times 5$  matrix, the value for the normal distribution is slightly higher at 42.37 (41.11 for the uniform distribution). For the  $3 \times 10$  matrix, the value for the normal distribution is greater at 35.18 (30.28 for the uniform distribution). And in the case of the  $5 \times 10$  matrix, the value for the normal distribution is 41.81 (40.21 for the uniform distribution).

The estimated average payoff values for the normal distribution are lower than those of the uniform distribution in the matrix scenarios where the Blue force has the higher number of options and the more capable forces. In the  $5 \times 3$  matrix, the value for the normal distribution is 56.46 (58.32 for the uniform distribution). For the  $10 \times 3$  matrix, the value for the normal distribution is 63.77 (68.65 for the uniform distribution). And for the  $10 \times 5$  matrix, the value for the normal distribution is 56.74 (59.72 for the uniform distribution).

The average standard deviation for the game payoffs using the normal distribution, 13.35, is less than that of the uniform distribution, 17.80. Additionally, the minimum outcome values for the normal distribution are greater than the outcome values for the uniform distribution. For the normal distribution, the minimum payoffs for the  $3 \times 3$ ,  $3 \times 5$  and  $3 \times 10$  matrices are 9.00, 10.00 and 2.00, respectively. In the case of the uniform distribution, the outcome payoffs decrease for the  $3 \times 3$ ,  $3 \times 5$  and  $3 \times 10$  matrices with values of 4.00, 2.00 and 0.00, respectively. Similarly, for the  $5 \times 3$ ,  $5 \times 5$  and  $5 \times 10$  matrices using the normal distribution, the minimum payoffs are 13.00, 13.00 and 11.00, respectively. In the case of the uniform distribution, the outcome payoffs decrease for the  $5 \times 3$ ,  $5 \times 5$  and  $5 \times 10$  matrices with values of 10.00, 6.00 and 3.00, respectively. Finally, for the  $10 \times 3$ ,  $10 \times 5$  and  $10 \times 10$  matrices using the normal distribution, the minimum game values are 28.00, 24.00 and 13.00, respectively, whereas the minimum values for the uniform distribution for the same matrices are 24.00, 17.00 and 0.00, respectively.

These results suggest that when a “tighter” distribution (truncated normal) is used to generate payoff values, the size of the effects of information superiority decreases.

## **V. CONCLUSIONS AND RECOMMENDATIONS**

### **A. OBSERVATIONS**

The conceptual framework of the Armed Force's Joint Vision serves as the basis for focusing the strengths of each individual service component to exploit the full array of available capabilities. This thesis provides several insights into one of the most important underlying concepts of the Joint Vision, decision superiority, and the value of information in defining force advantage and in determining how decisions and choices of actions may affect payoffs in battle. Numerous experiments were conducted and over 100,000 evaluations of matrix game simulations were performed in order to ensure that the estimated average payoffs are as accurate as possible over a variety of situations.

The first experiment observes the effects of opposing forces in battle when possessing asymmetric strategy choices. The experiment demonstrates that if opposing forces possess options with equivalent strategic capabilities, the payoff advantage is determined by the quantity of choices from which to choose. The payoff value increases linearly in favor of the force with the maximum number of choices. This suggests that flexible forces with more options have a significant advantage even when they do not possess advantages in payoff or information superiority.

The second experiment addresses the value of varying levels of information superiority between asymmetric opposing forces. The degree of advantage in payoff for the force with superior information is determined by the amount of choices and the quantity of bad information. When a force possesses significantly fewer strategic options, more superior information is required to assume the payoff advantage, however, information tends to have a greater value and provide a larger payoff gain for less capable forces with fewer choices. These results suggest that when a force has a significant disadvantage, there is substantial added value to information superiority. For a force having more flexibility and more strategies, significantly less information is required to affect an advantage in payoff, and superior information is less valuable and produces a smaller marginal gain for these more capable forces. This suggests that when a force already possesses an advantage, there is less benefit received from superior information.

Also in this experiment, a pattern exists in the results of the standard deviations. Looking across all cases, as the number of bad information choices increases for the Red force, the spread in payoffs increases.

The third experiment considers the effects of intelligence on the payoffs of asymmetric opposing forces with common levels of information. The results of the experiment demonstrate that intelligence provides the greatest payoff increase when a force possesses its maximum number of strategic options combined with the opposition also having its maximum number of choices. In the case where few options are available for opposing forces, intelligence provides minimal benefits to payoff advantage. This suggests that the more choices opposing forces have, the more important it is to have knowledge of an opponent's strategic intentions. Additionally, since intelligence provides the highest payoff in favor of the force with the most number of options, it is clear that when a force has a significant advantage, the more added benefit there might be from intelligence.

The fourth experiment examines the combined affects of both intelligence and information superiority, also known as information dominance, on the payoffs of opposing asymmetric forces. The results suggest that the presence of intelligence enhances the benefits of information superiority and allows forces to overcome their disadvantages at a faster rate, with less superior information. The experiment also shows that intelligence combined with information superiority produces the greatest benefit for the force with the largest disadvantage when its opposing force possesses significantly more capabilities. In contrast, the results show that little added benefit is received from the use of intelligence and information superiority by forces with significant advantages. The data also infers that the higher the number of available strategic choices a force has, the more added value the force may receive from information dominance.

The fifth experiment shows the effects on the payoffs of varying levels of superiority and inferiority in the capabilities of asymmetric forces. In the case of opposing forces when one force has increasingly superior capabilities, the use of the first superior option provides the largest payoff gain and levels off thereafter. In fact, the first superior option provides the highest advantage to the force with the fewest choices



against the most capable opposing force, therefore, making it possible for a force to overcome a severe disadvantage if it knows its most capable options. These results suggest that good information allows a force to utilize its better options even if only few exist. Therefore, looking across all cases of increasing superior capabilities, we see that if a force is aware of it, just one good option is all that it may need to gain the advantage in payoff.

In the case of opposing forces when one force has increasingly inferior capabilities, we see that the loss of force capability or the increase in inferiority reduces the estimated average payoff by the greatest margin for the force with the advantage. Meaning that when a force has its maximum number of choices available, while its opponent possesses its minimum number of options, the effects of inferiority have the largest impact on the payoff value of the battle. However, a force possessing a significant advantage with a high number of available choices, may reduce the rate at which inferiority impacts payoff loss by providing more options from which to choose, thereby, providing protection against the threat of bad information. The larger the advantage, the more the protection exists against just a few bad strategies, whereas if few options are present, inferiority has a higher negative impact on each strategy loss. These results suggest that a force with a significant advantage is more capable of defending against and preventing the loss in payoff due to a decrease in force capability. The greater the amount of choices, the more protection a force may gain against inferior options.

The sixth experiment uses normal distribution payoffs to compare the estimated average outcome values to those of the uniform distribution. This test suggests that the conclusions may be robust to other symmetric payoff distributions. The data also implies that when a “tighter” distribution (truncated normal) is used to generate payoff values, the size of the effects of information superiority decreases. These results show that different distributions have an impact on the outcome values.

The fundamental conclusion is that the benefits of various levels of information are dependent on numerous factors that affect a decision-maker’s choice of strategy and ultimately the payoff of battle. These experiments reflect the effect of knowledge and

capabilities on the likelihood of a successful outcome. Some of the results prove to be intuitive and others counter-intuitive, and it is the goal of this thesis to bring to the attention of the reader, the level of influence that the control of information has on the determination and decisiveness of victory.

Also implicit in these experiments is the notion that information superiority, dominance and their affects on payoffs result from dynamic interactions between two sides. This concept is intended to condition military commanders and decision-makers to understand that it is not satisfactory to calculate and consider the effects of our own strategic intentions, but an understanding of those of our opposition, may in some cases provide a more accurate comprehension of the course of battle and serve to further increase the probability of victory in our favor.

## **B. STUDY LIMITATIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH**

Extensions of the work previously performed by Bracken and Darilek on the value of information, and the conceptual dimensions as to how much information superiority is satisfactory to provide an advantage over one's opponent is necessary. Since Bracken and Darilek's research is limited to the assumption that each force either possesses correct or incorrect information and that the knowledge of the decisions of an opponent are either known or unknown, this research extends this concept by considering the value of information superiority, and its affects on the outcomes of decision-making as the information is varied between totally correct and incorrect. There exist numerous other methods of simulating the dynamics of varying the level information superiority. One example is to simulate the attrition of information. This is accomplished by deleting the payoff values within individual row or column strategies in order to represent either the loss of access to good information or a force's ability to prevent its opposition's use of that information.

In addition to varying the type and amount of information available to the two sides, the effects of opponents possessing different numbers of potential courses of action is also considered. Bracken and Darilek only use symmetric  $3 \times 3$ ,  $5 \times 5$  and  $10 \times 10$  matrices and this thesis simulates battles using  $3 \times 5$ ,  $3 \times 10$ ,  $5 \times 3$ ,  $5 \times 10$ ,  $10 \times 3$  and  $10 \times 5$  matrices in order to represent the probabilities of victory or payoffs for asymmetric forces

having different amounts of strategies and choices from which to choose. In order to more accurately model present military strategies, operations and battle scenarios, there would be even more accurate and applicable data if the size of the payoff matrices actually represent the number of choices that current forces in operation possess in several diverse scenarios. These strategies, instead of having numerical indices, should have labels with actual names of applicable strategies and the capabilities of these labels should be reflected in the payoffs. For example, for a Blue force row strategy representing airpower, the payoffs within that row should contain values that represent our threat capabilities at that particular time for that specific battle scenario.

In their research, Bracken and Darilek develop their payoffs using random numbers distributed uniformly between the boundaries of 0 and 100. In this following research, we also predominantly use a uniform distribution. Since the assumption that payoffs follow a uniform distribution is highly questionable, it is recommended that additional random number distributions be used to develop payoffs that are robust in exploring game scenarios that are representative of the dynamic interactions of actual combat.

The linear, quadratic and cubic regression models used to determine the local relationships between varying levels of information and battle outcomes ignore the fact that the payoffs are known to be bound between 0 and 100. An area for further study is to fit more realistic functional forms that use these constraints.

Since this thesis follows Bracken and Darilek by generating payoffs using random numbers to compute the average of 1,000 trials for each battle simulation, a recommendation is to use all 1,000 data points in fitting the models. This method will allow a more accurate identification of statistically significant effects than the ecological regression used in this thesis.

Our results suggest that flexible forces (i.e., those with more options) have a significant advantage even when they have no information or payoff advantage. A recommendation is to assess the effects of order statistics on the payoff values. For example, as the payoff value changes under various conditions, to what extent are these effects due to a force having more courses of actions or due to the fact that the range of a

sample of 10 options (more available courses of actions) is greater than that of a sample of 3 (few courses of actions)? And, how is the value of the game related to the order statistics of the sample.

In addition to two-person zero sum games, other types of games may provide insight into the value of varying levels of information for opposing forces. One such example is Blotto Games, named after the legendary Colonel Blotto, who was tasked with dividing his attacking force among several forts without knowing how the defenders were distributed.[Reference 6] In the generalization, each side has a certain total force that must be divided among the variable (n) “areas”. The payoff is a sum of payoffs in each area, and the payoff in each area depends only on the forces assigned to that area. An example application for Colonel Blotto games is the case where “areas” are specified communication and intelligence assets that provide the ability to obtain superior information and knowledge of opposing forces and their decisions.[Reference 6]

Sequential games provide an additional method of assessing the value of information between opposing forces since actual battle scenarios will include sequential decision-making where opposing force commander’s will be required to assess the game payoffs and perform further courses of action after their opponent responds to the initial round of strategy choices. Some of the benefits of sequential games are that decision-makers will be allowed to base decisions and choices of actions on tendencies and lessons learned from previous successful moves and victories as well as from memory of incorrect decisions and losses.

The listed recommendations for further research are only a sample of the wide range of topics that are available and necessary in providing further guidance and understanding for the role of Armed Force’s commanders and decision-makers in maintaining the assurance of information superiority, as we progress into the Information Age future of Joint Vision 2020.

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